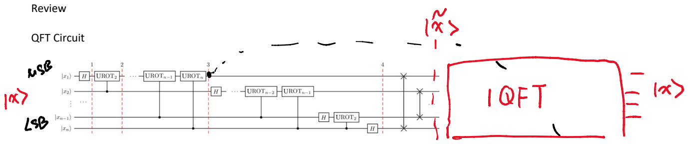
Quantum Phase Estimation (QPE)

Tuesday, October 20, 2020 9:21 PM

Review

QFT Circuit



$|\tilde{x}\rangle = QFT\,|x\rangle$ $\qquad x = x_1 x_2 \cdots x_n$ (MSB … LSB)

$= \frac{1}{\sqrt{N}} \bigotimes_{k=1}^{n} \left[|0\rangle + e^{\frac{i 2\pi x}{2^k}} |1\rangle \right]$

$= \frac{1}{\sqrt{N}} \left(|0\rangle + e^{i\frac{2\pi x}{2^1}} |1\rangle \right) \otimes \left(|0\rangle + e^{i\frac{2\pi x}{2^2}} |1\rangle \right) \otimes \left(|0\rangle + e^{i\frac{2\pi x}{2^3}} |1\rangle \right) \otimes \cdots \left(|0\rangle + e^{i\frac{2\pi x}{2^n}} |1\rangle \right)$

MSB $\qquad$ LSB

$e^{i\frac{2\pi}{2}x} \Leftrightarrow e^{i\theta 2^{n-1}}$

$\downarrow \qquad \downarrow$

$e^{i\frac{2\pi}{2^n}x} \Leftrightarrow e^{i\theta}$

$\Downarrow$

$\theta = \frac{2\pi}{2^n} x$

Single qubit QFT $\quad \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\frac{2\pi x}{2}}|1\rangle\right) \xrightarrow{IQFT} x$

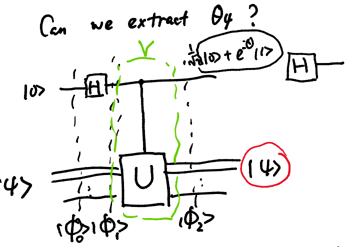
QPE

Problem: U quantum gate.

$U|\psi\rangle = e^{i\theta_\psi}|\psi\rangle$, assuming we can prepare $|\psi\rangle$

Can we extract θ_ψ ? $\qquad \theta = \frac{2\pi x}{2}$] example: $\theta = \pi$, $x = 1$

phase kick back



$|\phi_0\rangle = |0\rangle \otimes |\psi\rangle = |0\psi\rangle = |0\rangle|\psi\rangle$

$|\phi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|\psi\rangle = \frac{1}{\sqrt{2}}(|0\psi\rangle + |1\psi\rangle)$

$|\phi_2\rangle = U \cdot |\phi_1\rangle = \frac{1}{\sqrt{2}}(U \cdot |0\psi\rangle + U \cdot |1 \cdot \psi\rangle) = \frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + |1\rangle U|\psi\rangle)$

$= \frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + |1\rangle e^{i\theta}|\psi\rangle)$

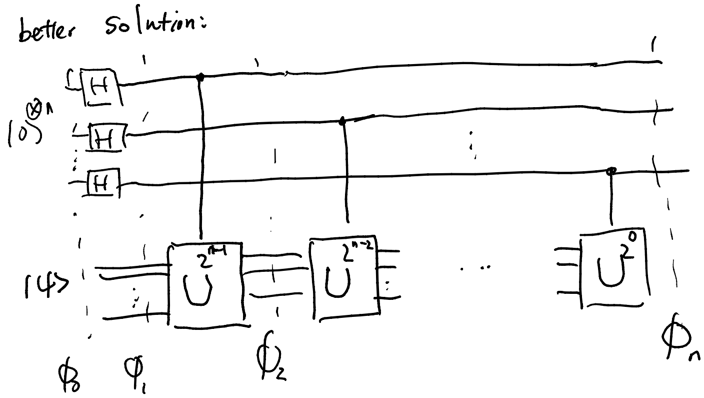
$= \frac{1}{\sqrt{2}}(|0\rangle + e^{i\theta}|1\rangle) \otimes |\psi\rangle$

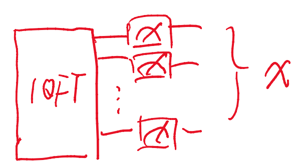
Z- rotation

$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ e^{i\theta} \end{bmatrix} = e^{i\theta}|1\rangle$

$\theta = 10^\circ$, prob(0) prob(1) = (0.9924, 0.007596)

A better solution:





$|\phi_0\rangle = |0\rangle^{\otimes n}|\psi\rangle$

$|\phi_1\rangle = \left(\frac{1}{\sqrt{2}}\right)^n (|0\rangle + |1\rangle)^{\otimes n} |\psi\rangle$

$$|\phi_0\rangle = |0\rangle\,|\psi\rangle$$

$$|\phi_1\rangle = \left(\frac{1}{\sqrt{2}}\right)^n \left(|0\rangle + |1\rangle\right)^{\otimes n} |\psi\rangle$$

$$U|\psi\rangle = e^{i\theta}|\psi\rangle \qquad U\cdot\left(U|\psi\rangle\right) = U\cdot e^{i\theta}|\psi\rangle = e^{i\theta}U|\psi\rangle = e^{i\theta}e^{i\theta}|\psi\rangle = e^{i2\theta}|\psi\rangle$$

$$U^{2^n}|\psi\rangle = e^{i\theta 2^n}|\psi\rangle$$

$$|\phi_2\rangle = \left(\frac{1}{\sqrt{2}}\right)^n \left(|0\rangle + e^{i\theta 2^{n-1}}|1\rangle\right)\left(|0\rangle+|1\rangle\right)^{n-1}|\psi\rangle$$

$$|\phi_n\rangle = \left(\frac{1}{\sqrt{2}}\right)^n \left(\left(|0\rangle + e^{i\theta 2^{n-1}}|1\rangle\right)\left(|0\rangle + e^{i\theta 2^{n-2}}\right)\cdots\left(|0\rangle + e^{i\theta 2^{0}}|1\rangle\right)\right)|\psi\rangle$$

$$\Downarrow$$

$$|QFT$$

$$\Downarrow$$

$$x \longrightarrow \underline{\theta = \frac{2\pi}{2^n}x}$$

https://algassert.com/quirk#circuit=%7B%22cols%22%3A%5B%5B%22H%22%2C%22H%22%2C%22H%22%2C%22X%22%5D%2C%5B%22%E2%80%A2%22%2C1%2C1%2C%22Z%22%5D%2C%5B1%2C%22%E2%80%A2%22%2C1%2C%22Z%5E%C2%BD%22%5D%2C%5B1%2C1%2C%22%E2%80%A2%22%2C%22Z%5E%C2%BC%22%5D%2C%5B%22X%22%2C1%2C%22%E2%80%A2%22%5D%2C%5B%22%E2%80%A2%22%2C1%2C%22X%22%5D%2C%5B%22X%22%2C1%2C%22%E2%80%A2%22%5D%2C%5B%22QFT%E2%80%A03%22%5D%2C%5B%22Amps3%22%5D%5D%7D

From <https://algassert.com/quirk#circuit={%22cols%22:[[%22H%22,%22H%22,%22H%22,%22X%22],[%22%E2%80%A2%22,1,1,%22Z%22],[1,%22%E2%80%A2%22,1,%22Z^%C2%BD%22],[1,1,%22%E2%80%A2%22,%22Z^%C2%BC%22],[%22X%22,1,%22%E2%80%A2%22],[%22%E2%80%A2%22,1,%22X%22],[%22X%22,1,%22%E2%80%A2%22],[%22QFT%E2%80%A03%22],[%22Amps3%22]]}>