Principal Concepts behind IBM Q

D. Stancil, Quantum Computing CSC591/ECE592, 24 Sept. 2018

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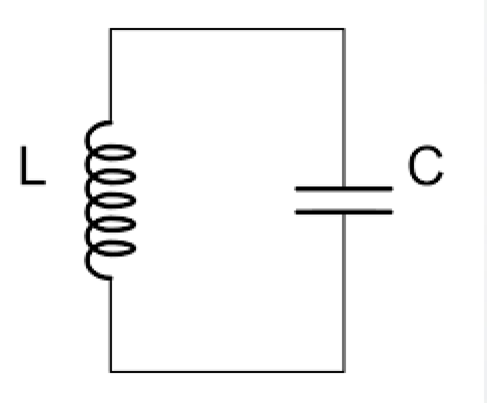
Outline

- How to make qubits with circuits
- Superconductivity
- Josephson junction and nonlinear LC circuits
- SQUID: Tunable nonlinear inductance
- Transmon
- Rabi oscillations
- Implementation of X,Y,Z,H gates
- Entangled transmons and controlled gates
- IBM examples

(Subset of) Requirements for Quantum Computer

- Physical system with two uniquely addressable states
- Ability to implement arbitrary rotations on the Bloch sphere
- Ability to measure the state of a cubit
- Ability to entangle two qubits

Qubit possibility: LC Resonant Circuits?



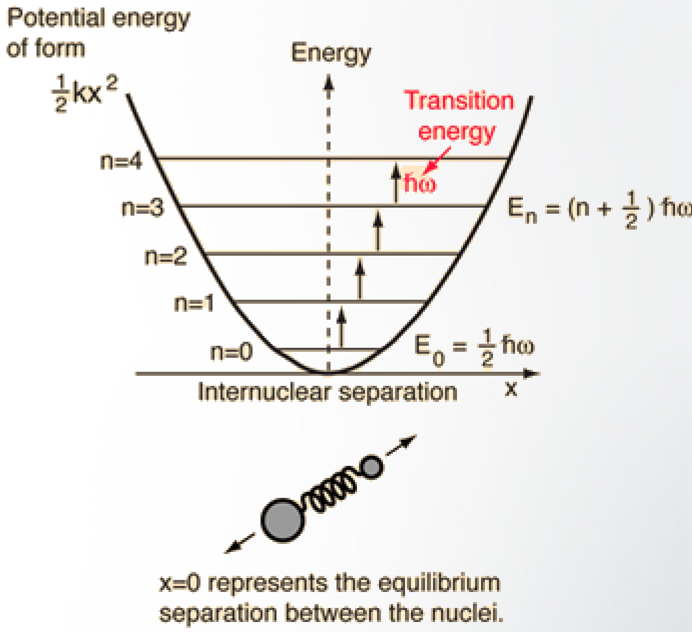
- In classical, linear circuit theory, the natural solution for the current is

$$i(t) = I_0 \cos \omega_0 t, \quad \omega_0 = 1/\sqrt{LC}$$

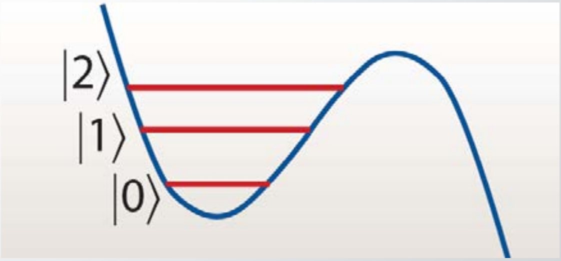
- The current can have any amplitude, independent of the frequency
- Energy is stored alternately in the electric field of the capacitor and the magnetic field of the inductor, and can have any value $U \sim I_0^2$
- In reality, the stored energy is quantized $U = \hbar\omega_0 \left(n + \frac{1}{2} \right)$
- Could we use two of these states for a qubit, say n=0 and n=1?

Nonlinear LC Circuits

- Problem: energy difference between 0,1 is the same as energy difference between all other states n,n+1
  - No way to address specific states
- Solution: if either L or C were nonlinear (i.e., their values depended on the magnitude of the current or voltage), then the energy levels would no longer be equally spaced!
  - If the energy difference between n=0 and n=1 was different from the energy difference from all other states, we can selectively address this particular transition by tuning the frequency of the applied excitation



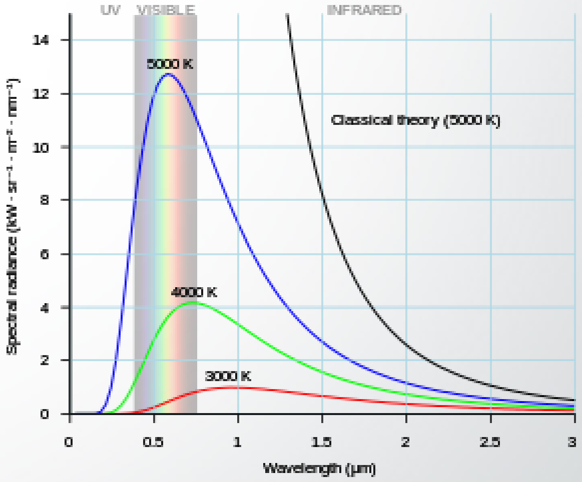
Graphic: http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/hosc.html



Graphic: Clarke & Wilhelm

RF Frequency

- The fact that we are talking about circuits suggests we are talking about RF rather than optical frequencies!
- Problem: any physical mass at finite temperature will emit electromagnetic radiation that depends on its temperature (Black body radiation)
  - We want the energy difference between qubit states to be large compared to thermal radiation
  - Highest frequency for widespread, economical instrumentation ~ 6 GHz (owing, e.g., to WIFI, etc.)
  - From kT=hf, the temperature corresponding to 6 GHz is 0.29K
  - Operating temperature must be much less than 0.3K!
- Solution: IBM Q systems operate at a temperature of about 15 mK using dilution refrigeration



Graphic: Wikipedia

Superconductivity

- At such low temperatures, metals such as Al and Nb become superconductors
  - At low temperatures, an attractive force between electrons appears
  - When this force gets sufficiently strong compared to thermal vibrations, electrons bind together into “Cooper pairs” with spin 1 and charge 2q
  - Cooper pairs form a macroscopic quantum state enabling charge to move without scattering or loss, resulting in superconductivity
- Makes it possible to make extremely low-loss RF transmission lines
- Makes it possible to realize a nonlinear inductor using a Josephson junction

A note to Physicists: Cooper Pairs and Superconductivity

- Spin ½ particles are “Fermions”
  - Fermions obey the Pauli exclusion principle: no two can be in the same state
  - Electrons are Fermions
- Spin 1 particles are “Bosons”
  - Bosons do not obey the Pauli exclusion principle: you can have as many in a state as you want
  - Photons are Bosons
- In a superconductor, an effective attractive interaction between electrons causes them to be loosely bound together and act like a single spin 1 particle: “Cooper Pair”
- Since Cooper pairs are spin 1, they act like Bosons, and you can have multiple Cooper pairs in the same state
- All of the Cooper pairs in a macroscopic sample can be in the same coherent state

Cooper Pairs are the result of the Electron-Phonon interaction in the theory of Bardeen, Cooper, and Schreifer (BCS Theory)

- Electrons normally repel one another, but are attracted to ions in the crystal lattice
- If the ions are pulled slightly toward an electron, from a distance it can appear as though there is a net positive charge, attracting another electron

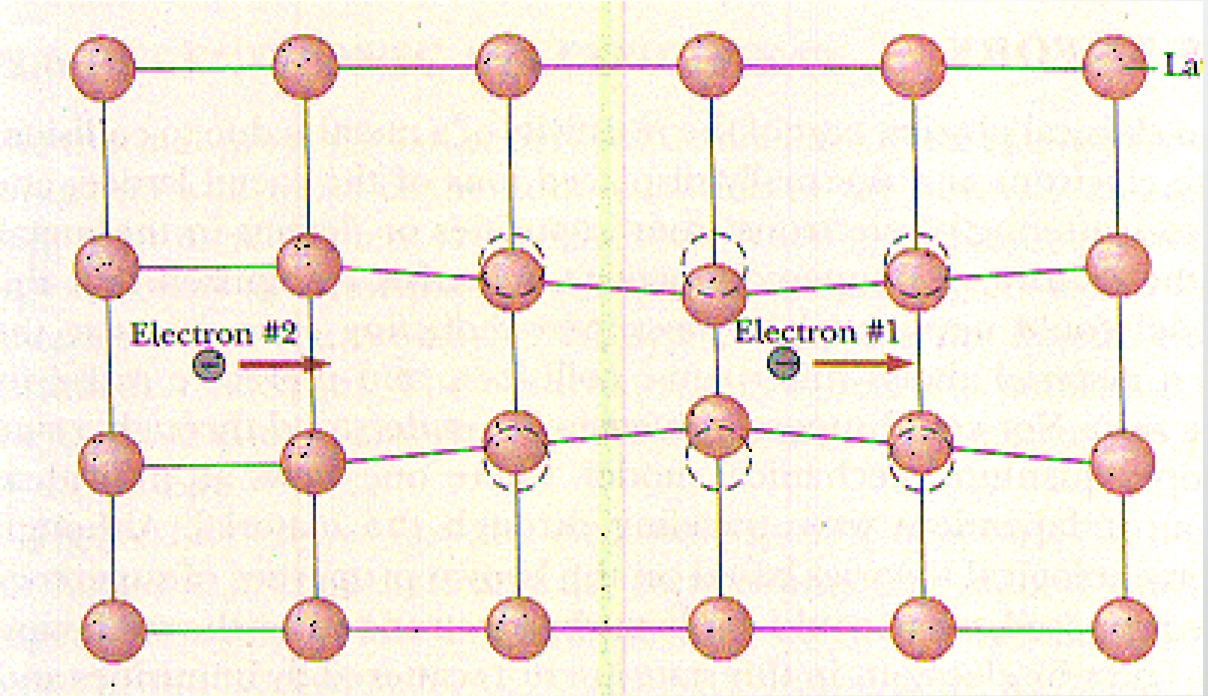
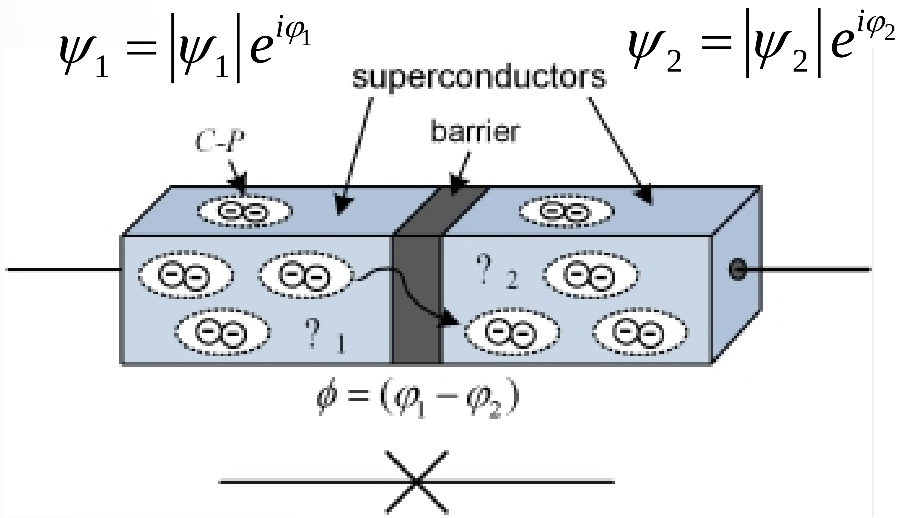
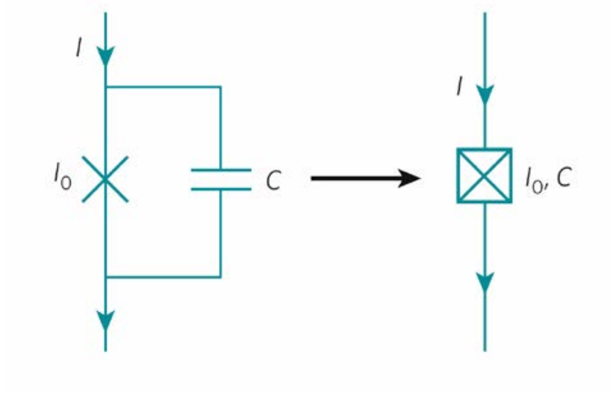


Image from Quora

Josephson tunnel junction



Circuit Symbols



Graphic: Clarke & Wilhelm

- Two superconductors separated by a thin insulating layer
- Wave functions for superconducting Cooper pairs decay exponentially in the insulating layer
- If the layer is thin enough to allow appreciable tunneling, then phases are no longer independent but are related to each other through the size of the tunneling current

Graphic: Ph.D. thesis: Vratislav Michal

Josephson Junction as nonlinear inductor

$$\varphi = \varphi_2 - \varphi_1$$

$$I = I_c \sin\varphi$$

$$V = \frac{\Phi_0}{2\pi}\frac{d\varphi}{dt}, \quad \Phi_0 = \frac{h}{2q} \text{ is the flux quantum}$$

$$\frac{dI}{dt} = I_c \cos\varphi \frac{d\varphi}{dt}$$

$$= I_c \cos\varphi \frac{2\pi V}{\Phi_0}$$

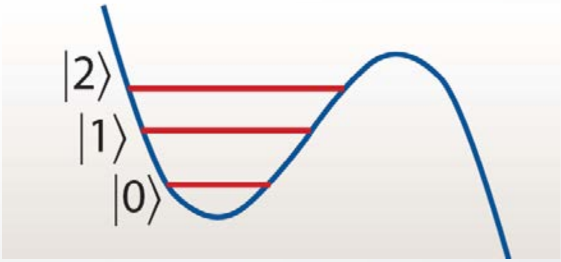
$$\Rightarrow$$

$$V = \frac{\Phi_0}{2\pi I_c \cos\varphi}\frac{dI}{dt}$$

$$= \frac{\Phi_0}{2\pi I_c \sqrt{1-\sin^2\varphi}}\frac{dI}{dt}$$

$$= \frac{\Phi_0}{2\pi I_c \sqrt{1-(I/I_c)^2}}\frac{dI}{dt} = L_{eff}(I)\frac{dI}{dt}$$

- Effective inductance depends on the current
- Looks like a non-linear inductor: origin of *anharmonicity*: spacing between energy levels is not the same
  - *Enables the individual addressing of a single pair of states*
  - *In contrast, in a linear circuit, all states are equally spaced*



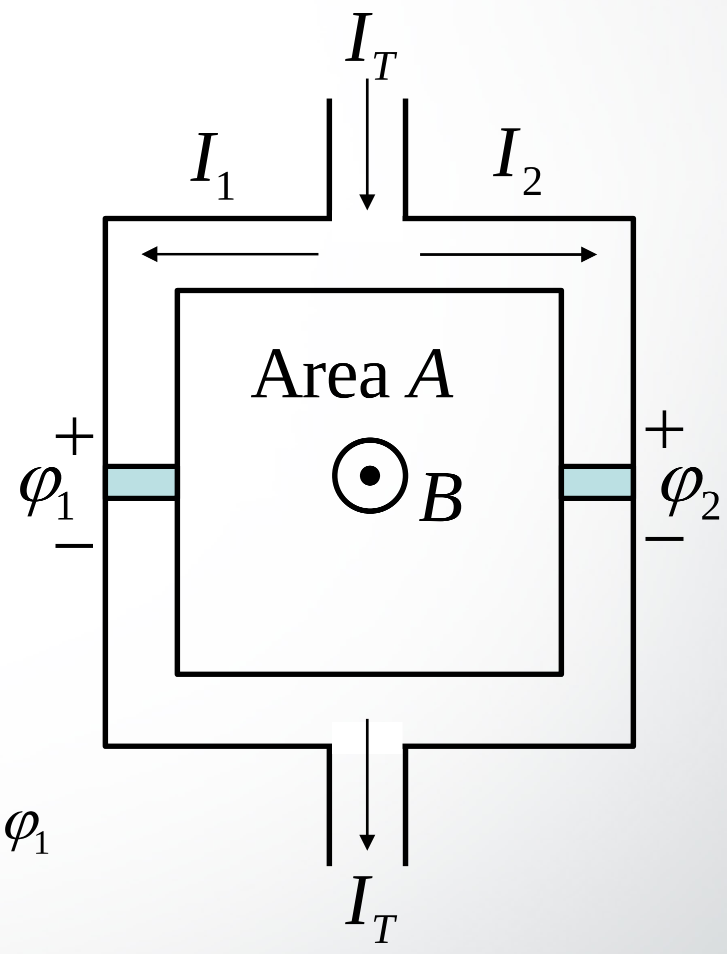
Superconducting Quantum Interference Device (SQUID)

- Parallel Josephson Junctions
- Magnetic field induces circulating current
- Simple analysis: neglect inductance of loop, assume both JJs are identical

$$I_T = I_1 + I_2 = I_c \sin\varphi_1 + I_c \sin\varphi_2$$

$$\varphi_2 = \varphi_1 + 2\pi\Phi / \Phi_0$$

$$\Phi = BA$$



- The second equation comes from integrating the canonical momentum around the loop (See e.g., Van Duzer & Turner)
- If the total current is zero:
- Thus applying a magnetic field will induce a current, and consequently tune the inductance

$$\sin\varphi_2 = -\sin\varphi_1$$

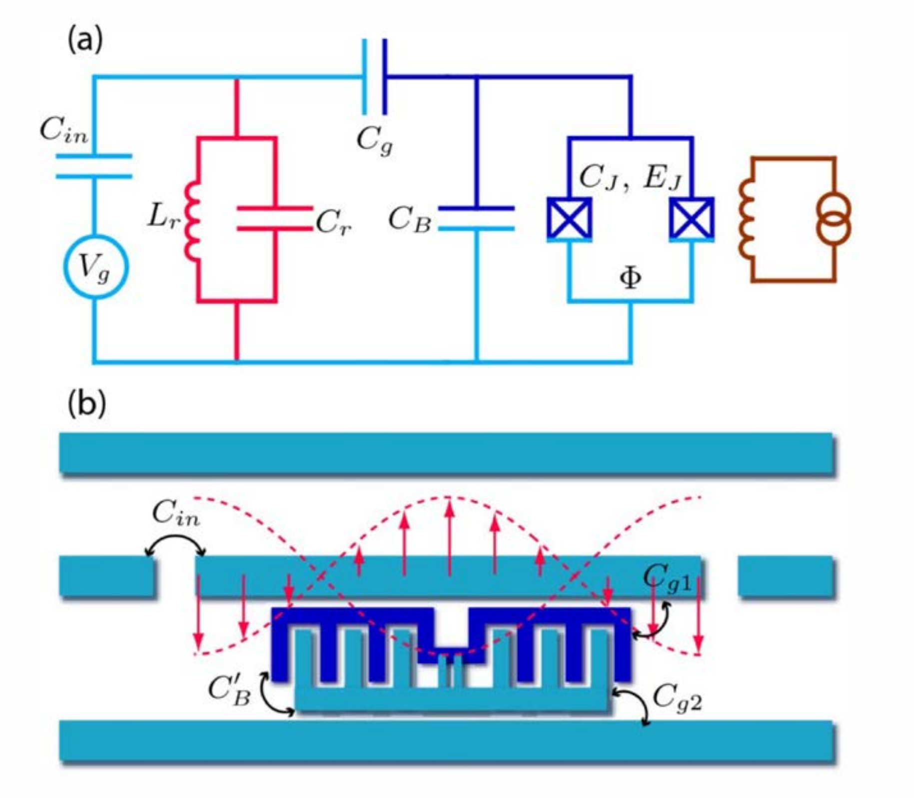
$$\sin\left(\varphi_1 + 2\pi\Phi / \Phi_0\right) = -\sin\varphi_1$$

$$\varphi_1 + 2\pi\Phi / \Phi_0 \approx -\varphi_1$$

$$\varphi_1 = -\pi\Phi / \Phi_0 = -\varphi_2$$

Transmon

- Adds extra capacitance by adding $\lambda/20$ transmission lines to either side of a pair of Josephson Junctions forming a SQUID
  - Extra capacitance reduces noise from charge fluctuations
  - Resonant frequency determined by JJ inductance and shunting capacitance
  - SQUID allows flux tuning
- A Single Josephson Junction can be used for a fixed-frequency qubit



Koch, et al (not to scale)

Transmon Designs

Fixed Frequency Qubits

Coplanar microwave resonator

Flux Tunable Qubits

Current control line

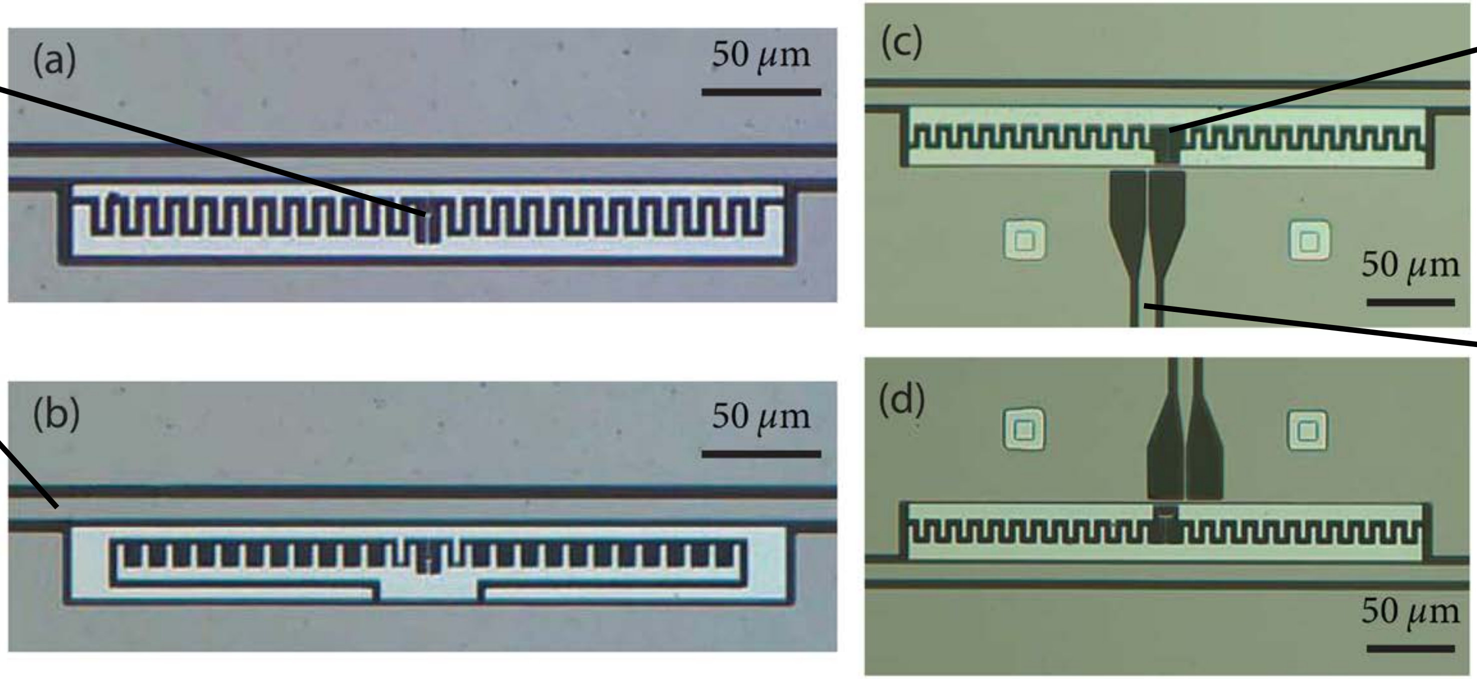
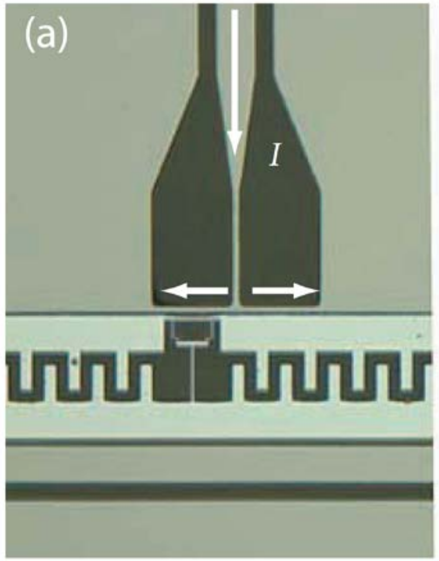


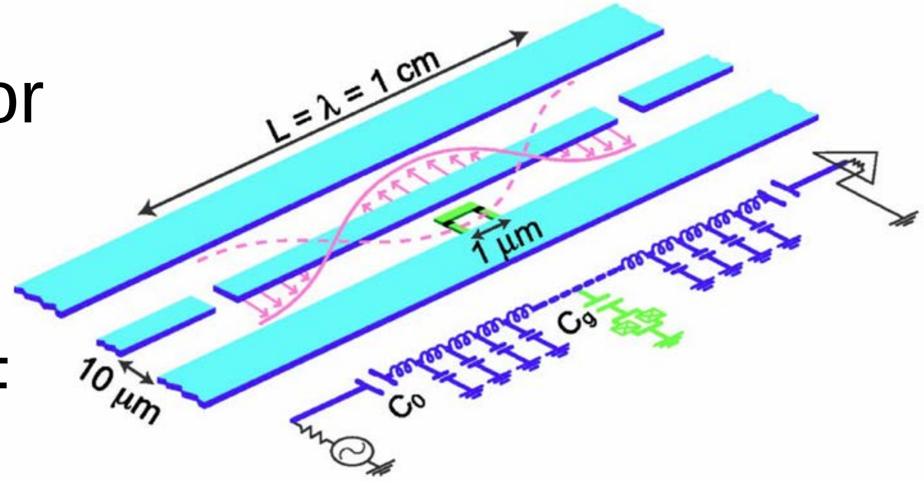
Figure 5.3: Optical images of different transmon designs. (a) Standard transmon design employed in cQED157 and on one of the qubits in cQED187. (b) Balanced transmon design used in one of the qubits in cQED187. (c) and (d) Transmon designs incorporating flux bias lines. A slightly different transmon SQUID loop design is necessary to accommodate the flux bias lines entering from the (c) bottom of the chip or from the (d) top of the chip, while preserving the same double-angle evaporation procedure.



Chow, PhD Thesis

Coupling to a Transmon

- Co-planar microstrip resonator formed by gaps in center conductor
- Important to properly choose resonator frequency with respect to transmon frequency (more to come)
- Control is achieved by injecting an RF signal from one end
- Readout is achieved by looking at either the transmitted or reflected signal



Blais, et al

Rabi Oscillations

Rabi Oscillations

- When a two-level system is coupled to a driving field at precisely the frequency corresponding to the energy difference between the states, the system will oscillate between the two states at the Rabi frequency

$$|\psi\rangle = c_g(t)|0\rangle + c_e(t)|1\rangle$$

if $c_g(0)=1,\ c_e(0)=0,$ then

$$P_g(t) = |c_g(t)|^2 = \frac{1}{2}\left(1+\cos\Omega_R t\right)$$

$$P_e(t) = |c_e(t)|^2 = \frac{1}{2}\left(1-\cos\Omega_R t\right)$$

$$\Omega_R = \frac{2}{\hbar}\langle 1|H_I|0\rangle$$

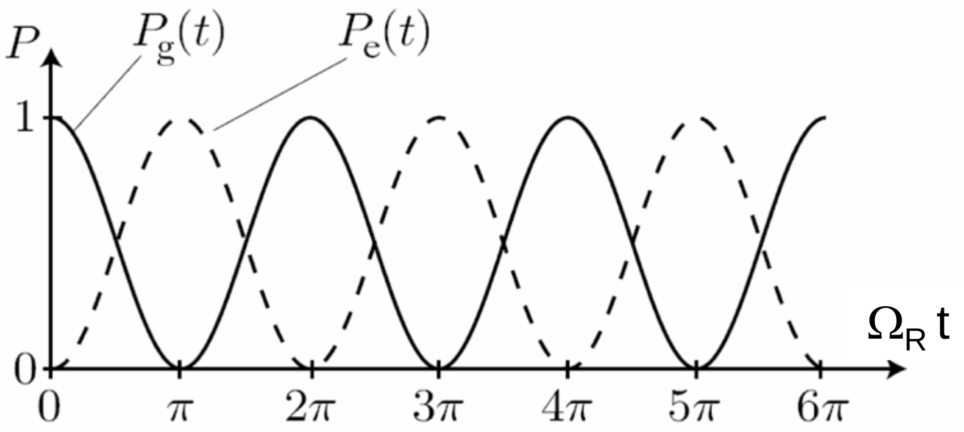


Figure 51: Time evolution of the probability $P_g(t)$ and $P_e(t)$ to find the atom in the ground (solid) and excited (dashed) state, respectively. [from D.A. Steck *Quantum and Atom Optics*]

Schrodinger Equation

$$i\hbar\partial_t|\psi\rangle = H|\psi\rangle, \quad \partial_t \equiv \frac{\partial}{\partial t}$$

- *H* is an expression for the total energy of the system (kinetic + potential) and is called the *Hamiltonian*
- For two orthogonal states:

$$i\hbar\partial_t|1\rangle = H_0|1\rangle = E_1|1\rangle$$

$$i\hbar\partial_t|2\rangle = H_0|2\rangle = E_2|2\rangle$$

- What if we introduce a perturbation that weakly couples states 1 & 2?

$$i\hbar\partial_t|1\rangle = H_0|1\rangle + V_{12}|2\rangle$$

$$i\hbar\partial_t|2\rangle = H_0|2\rangle + V_{21}|1\rangle$$

Schrodinger Equation for 2x2 coupled system

$$i\hbar\partial_t |1\rangle = H_0 |1\rangle + V_{12} |2\rangle$$

$$i\hbar\partial_t |2\rangle = H_0 |2\rangle + V_{21} |1\rangle$$

- This can be written

$$i\hbar\partial_t \begin{bmatrix} |1\rangle \\ |2\rangle \end{bmatrix} = \begin{bmatrix} E_1 & 0 \\ 0 & E_2 \end{bmatrix} \begin{bmatrix} |1\rangle \\ |2\rangle \end{bmatrix} + \begin{bmatrix} 0 & V_{12} \\ V_{21} & 0 \end{bmatrix} \begin{bmatrix} |1\rangle \\ |2\rangle \end{bmatrix} = H \begin{bmatrix} |1\rangle \\ |2\rangle \end{bmatrix}$$

$$H = \frac{1}{2}(E_1 + E_2)\sigma_0 + \frac{1}{2}(E_1 - E_2)\sigma_z + V\sigma_x, \quad \text{assuming } V_{12} = V_{21}$$

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \ \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Formulations of Quantum Mechanics

- "Schrodinger Picture:" all time dependence is in the wave function or state vector
- "Heisenberg Picture:" all time dependence is in the Hamiltonian operator
- "Interaction Picture:" hybrid in which some time dependence is in both the operator and the state vector
  - Particularly useful when considering a small perturbation to a solved system: express solution to perturbed system in terms of solutions to the unperturbed system

Interaction Picture

- Let us express the Hamiltonian as a sum of two terms, the larger of which is time independent and for which solutions are known, and a smaller perturbation that contains all of the time dependence: $H = H_0 + V(t)$
- In the Schrodinger picture: $i\hbar\partial_t \left|\psi(t)\right\rangle_S = H\left|\psi(t)\right\rangle_S$
- The transformation for the state vector into the Interaction picture:

$$\left|\psi(t)\right\rangle_I = e^{iH_0t/\hbar}\left|\psi(t)\right\rangle_S$$

- Equation of motion:

$$\begin{aligned} i\hbar\partial_t \left|\psi(t)\right\rangle_I &= i\hbar\partial_t\left(e^{iH_0t/\hbar}\left|\psi(t)\right\rangle_S\right) \\ &= e^{iH_0t/\hbar}\left(i\hbar\partial_t - H_0\right)\left|\psi(t)\right\rangle_S \\ &= e^{iH_0t/\hbar}\left(H_0 + V - H_0\right)\left|\psi(t)\right\rangle_S \end{aligned}$$

$$\begin{aligned} i\hbar\partial_t \left|\psi(t)\right\rangle_I &= e^{iH_0t/\hbar}V\left|\psi(t)\right\rangle_S \\ &= \underbrace{e^{iH_0t/\hbar}Ve^{-iH_0t/\hbar}}_{V_I(t)}\underbrace{e^{iH_0t/\hbar}\left|\psi(t)\right\rangle_S}_{\left|\psi(t)\right\rangle_I} \end{aligned}$$

- Only depends on time-dependent perturbation: $\boxed{i\hbar\partial_t \left|\psi(t)\right\rangle_I = V_I(t)\left|\psi(t)\right\rangle_I}$

Solution to Equation of Motion

- Construct an interaction picture solution by adding up eigenstates of the unperturbed Hamiltonian but with time-dependent coefficients:

$$\left|\psi(t)\right\rangle_I = \sum_n c_n(t)\left|n\right\rangle$$

- Substitute into equation of motion:

$$i\hbar\partial_t \sum_n c_n \left|n\right\rangle = e^{iH_0t/\hbar} V e^{-iH_0t/\hbar} \sum_n c_n \left|n\right\rangle$$

$$i\hbar \sum_n \dot{c}_n \left|n\right\rangle = e^{iH_0t/\hbar} V \sum_n c_n e^{-iE_nt/\hbar} \left|n\right\rangle$$

$$i\hbar \sum_n \dot{c}_n \underbrace{\left\langle m|n\right\rangle}_{\delta_{mn}} = \underbrace{\left\langle m\right| e^{iH_0t/\hbar}}_{\left\langle m\right| e^{iE_mt/\hbar}} V \sum_n c_n e^{-iE_nt/\hbar} \left|n\right\rangle$$

$$i\hbar\dot{c}_m = \sum_n V_{mn} e^{i\omega_{mn}t} c_n,$$

$$V_{mn} = \left\langle m\right|V\left|n\right\rangle, \quad \omega_{mn} = \left(E_m - E_n\right)/\hbar = -\omega_{nm}$$

Application to 2-Level System

- Recall: $$H = \frac{1}{2}(E_1 + E_2)\sigma_0 + \frac{1}{2}(E_1 - E_2)\sigma_z + V\sigma_x$$
- Choose zero of energy as the average, and *V* caused by sinusoidal RF signal: $$H = -\frac{1}{2}\hbar\omega_{21}\sigma_z + V_0\cos\left(\omega^{rf}t + \phi\right)\sigma_x$$
- Equations of motion $i\hbar\dot{c}_n = \sum_n V_{mn} e^{i\omega_{mn}t} c_n$ become:

$$i\hbar\dot{c}_1 = V_0\cos\left(\omega^{rf}t + \phi\right)e^{i\omega_{12}t}c_2$$

$$i\hbar\dot{c}_2 = V_0\cos\left(\omega^{rf}t + \phi\right)e^{i\omega_{21}t}c_1$$

$$i\hbar\dot{c}_1 = \frac{V_0}{2}\left(e^{i\left(\omega^{rf}t+\phi\right)} + e^{-i\left(\omega^{rf}t+\phi\right)}\right)e^{-i\omega_{21}t}c_2$$

$$i\hbar\dot{c}_2 = \frac{V_0}{2}\left(e^{i\left(\omega^{rf}t+\phi\right)} + e^{-i\left(\omega^{rf}t+\phi\right)}\right)e^{i\omega_{21}t}c_1$$

Solution in the Rotating Wave Approximation

- Assume the energy associated with the RF frequency is close to the transition energy between the two states: $\omega^{rf} = \omega_{21} + \Delta, \quad |\Delta| \ll \omega_{21}$
- Then keep only the low-frequency components to the solution (Rotating Wave Approximation):

$$i\hbar\dot{c}_1 = \frac{V_0}{2} e^{i\left(\omega^{rf}-\omega_{21}\right)t+i\phi} c_2 = \frac{V_0}{2} e^{i\Delta t+i\phi} c_2$$

$$i\hbar\dot{c}_2 = \frac{V_0}{2} e^{-i\left(\omega^{rf}-\omega_{21}\right)t-i\phi} c_1 = \frac{V_0}{2} e^{-i\Delta t-i\phi} c_1$$

- Substituting the first equation into the second gives a second-order Diff. Equation for c_1:

$$\ddot{c}_1 - i\Delta\dot{c}_1 + \left(\frac{V_0}{2\hbar}\right)^2 c_1 = 0$$

- If the system starts off in state 1, then it is easily verified that the solution is

$$c_1(t) = Ae^{i\Delta/2}\cos\left(\Omega t/2\right)$$

$$c_2(t) = i\frac{2A\hbar}{V_0} e^{-i\Delta/2-i\phi}\left(i\frac{\Delta}{2}\cos\left(\Omega t/2\right) - \frac{\Omega}{2}\sin\left(\Omega t/2\right)\right)$$

$$\Omega = \sqrt{\Delta^2 + \left(\frac{V_0}{\hbar}\right)^2}$$

$$|c_1|^2 + |c_2|^2 = 1 \;\Rightarrow\; A = \left(1 + \left(\Delta\hbar/V_0\right)^2\right)^{-1/2}$$

Driving at Resonance: Rabi Oscillations

- If the RF driving frequency corresponds to the energy difference between the two states, then

$$\Delta = 0, \ \ \omega^{rf} = \omega_{21}$$

- The coefficients then become

$$c_1(t) = \cos\left(\Omega_R t / 2\right)$$
$$c_2(t) = -ie^{-i\phi}\sin\left(\Omega_R t / 2\right)$$
$$\Omega_R = V_0 / \hbar$$

- Let state 1 correspond to the ground state, and state 2 to the excited state. The probability of finding the system in each state is given by

$$\left|c_g(t)\right|^2 = \left(\cos\left(\Omega_R t / 2\right)\right)^2 = \frac{1}{2}\left(1 + \cos\Omega_R t\right)$$
$$\left|c_e(t)\right|^2 = \left(\sin\left(\Omega_R t / 2\right)\right)^2 = \frac{1}{2}\left(1 - \cos\Omega_R t\right)$$

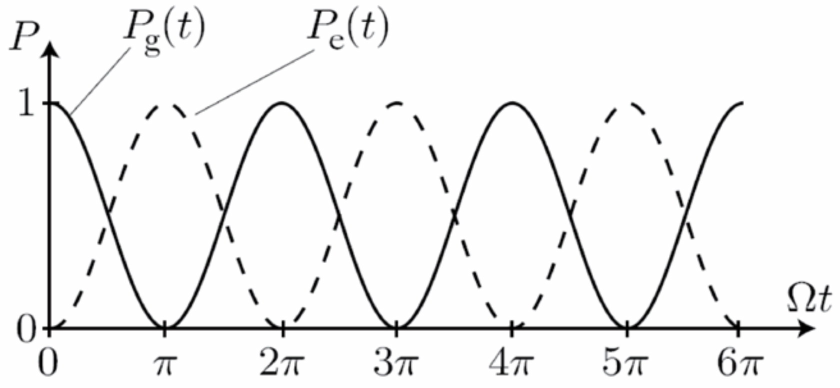
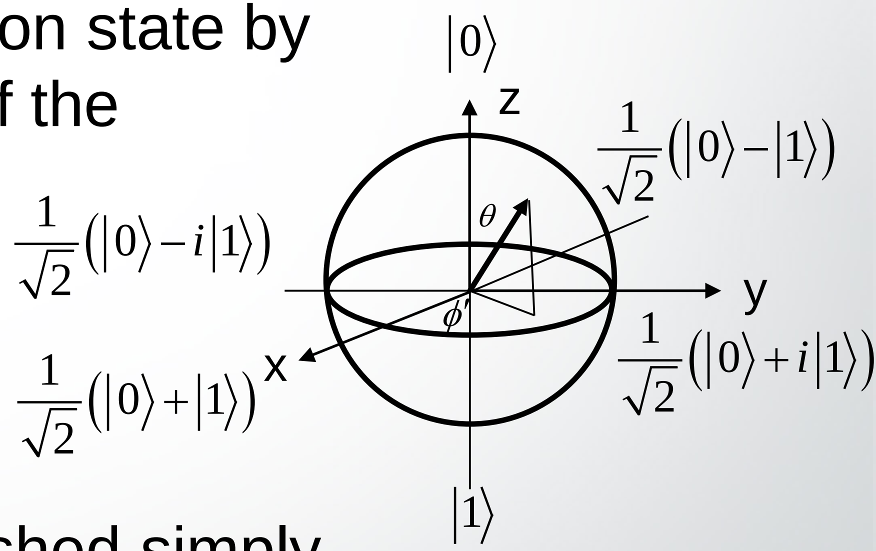


Figure 51: Time evolution of the probability $P_g(t)$ and $P_e(t)$ to find the atom in the ground (solid) and excited (dashed) state, respectively. [from D.A. Steck *Quantum and Atom Optics*]

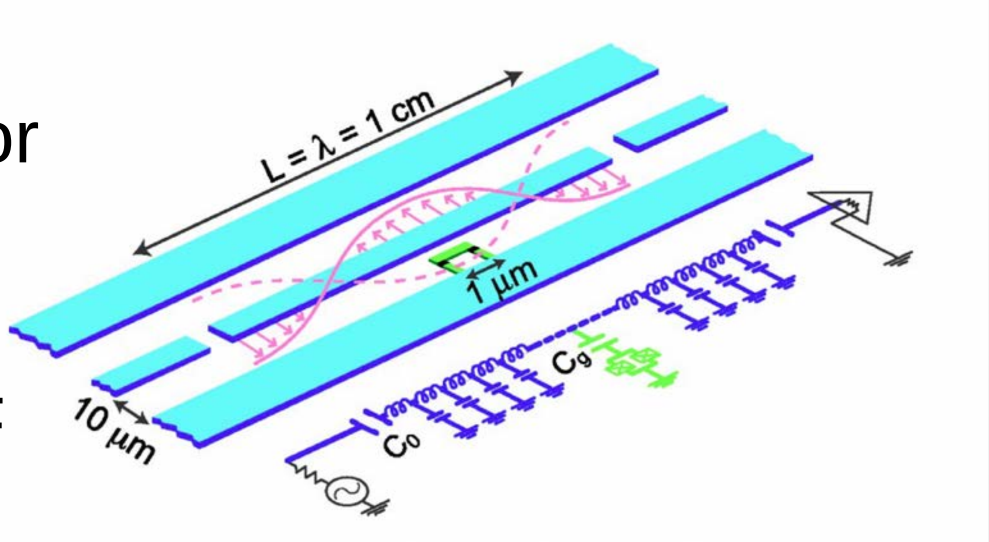
Realizing Gates using Rabi Oscillations

- Recall that when driven at resonance $$c_1(t) = \cos(\Omega_R t/2)$$ $$c_2(t) = -ie^{-i\phi}\sin(\Omega_R t/2)$$
- "π-pulse": $\Omega_R t = \pi$ inverts the state
- "π/2-pulse": $\Omega_R t = \pi/2$ creates equal superposition of states
- Key point: you can flip a state or create a superposition state by controlling the pulse length & controlling the phase of the excitation
- X gate: pi-pulse with $\phi = -\pi/2$
- Y gate: pi-pulse with $\phi = \pi$
- Hadamard gate: pi/2-pulse with $\phi = -\pi/2$
- Z gate: Rotations around the z axis can be accomplished simply by resetting the phase reference of the RF drive! Result: zero time and zero error



Coupling to a Transmon

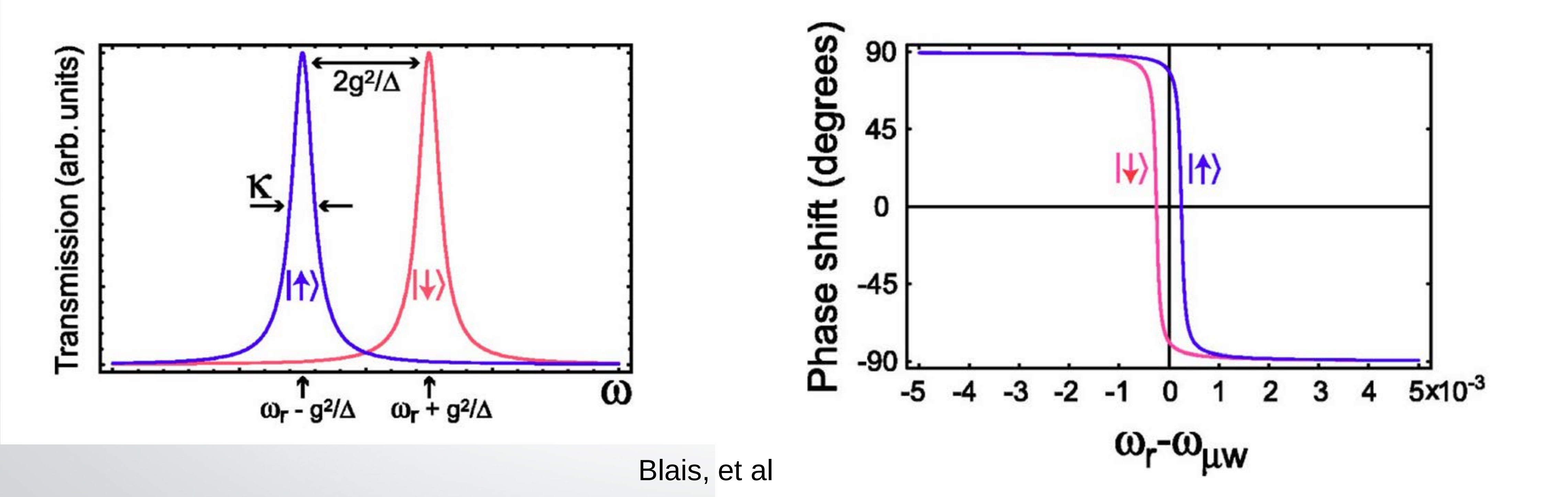
- Co-planar microstrip resonator formed by gaps in center conductor
- Important to properly choose resonator frequency with respect to transmon frequency (more to come)
- Control is achieved by injecting an RF signal from one end
- Readout is achieved by looking at either the transmitted or reflected signal



Blais, et al

Control and Read-out

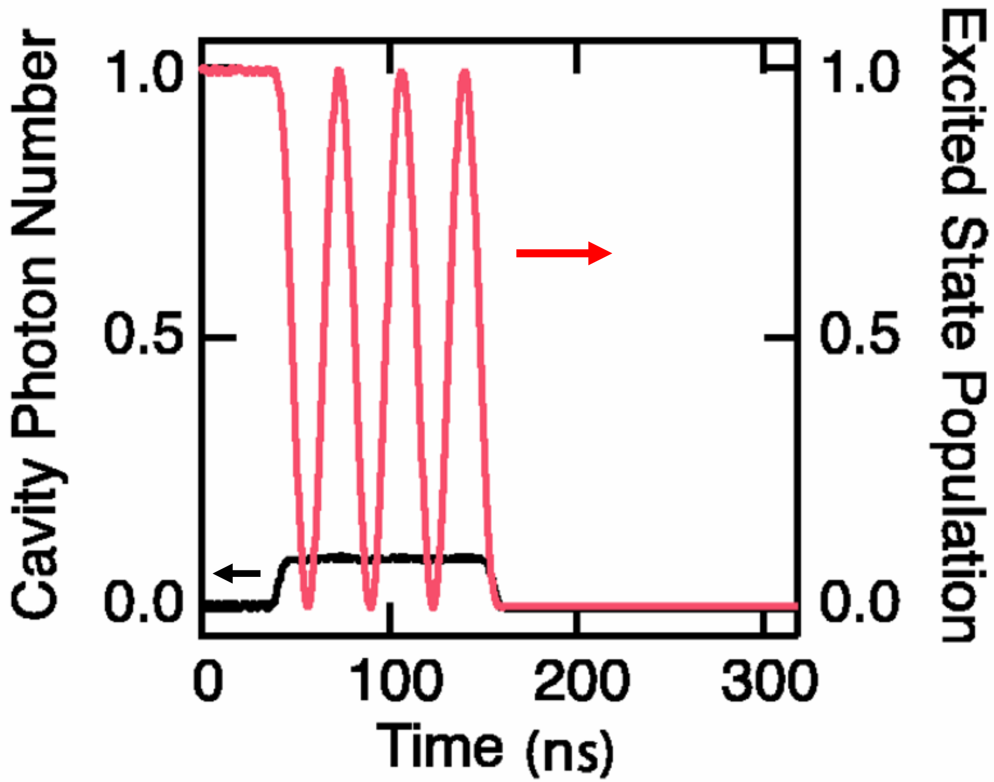
- When the microstrip resonator is detuned from the transmon frequency, interaction with the transmon splits the resonator response into two modes, depending on the transmon state
- Sending in a pulse near ω_r enables you to read-out the state either from the phase, or the amplitude
- Sending in a pulse detuned from the microstrip resonator but tuned to the qubit frequency rotates the state, but does not make a measurement (there is no information about the state in the reflected signal, the resonator is so far off resonance)



Blais, et al

Control Pulse simulation

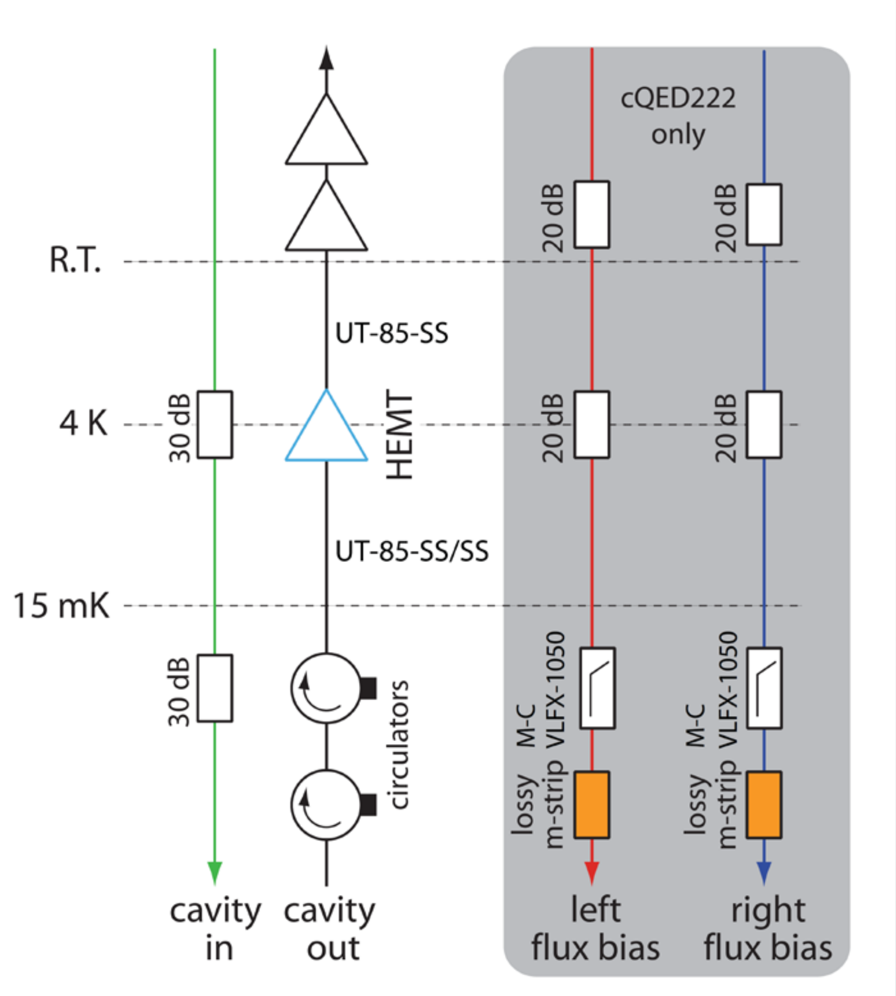
- Control signal turned on for 7 pi pulses then turned off
- The state rotates between the excited and ground states
- The cavity photon level is small since the cavity is detuned from resonance



Blais, et al

RF Electronics

- Cooled attenuators keep room temperature noise from the qubit
  - But this means you must send a very strong signal!
- Similarly, cooled circulators keep room temperature noise from the qubit without attenuating the signal

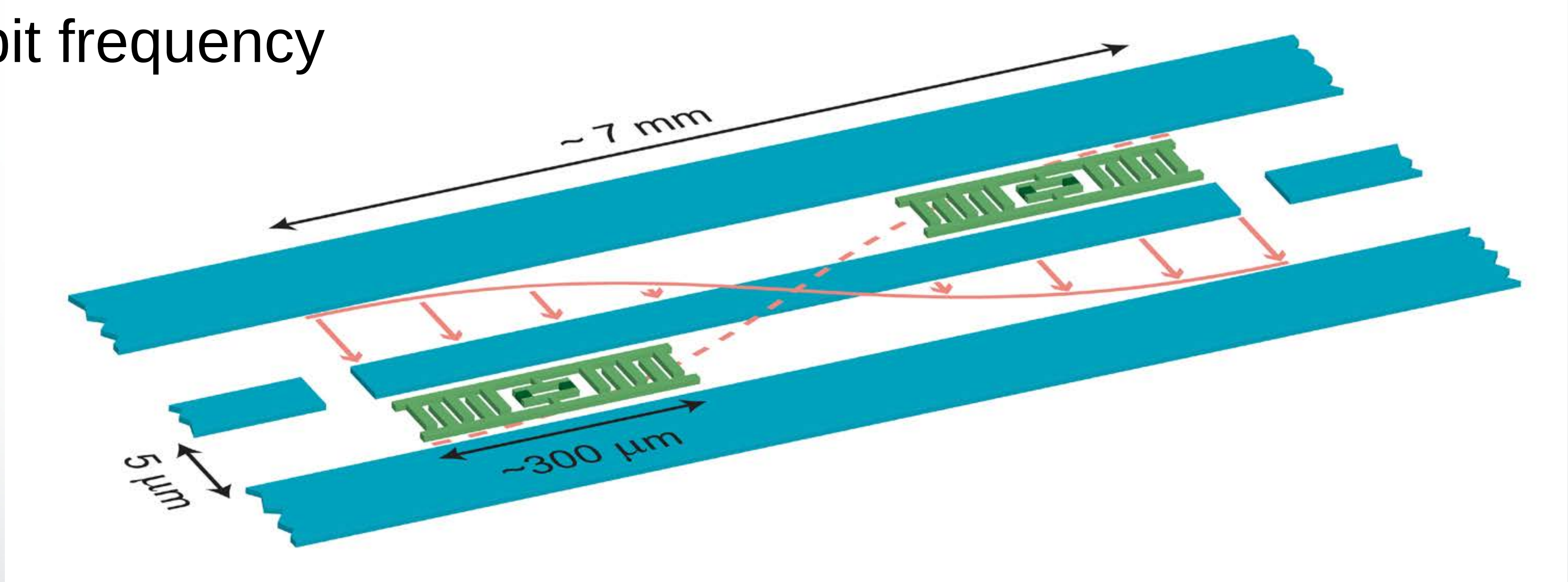


Chow, PhD Thesis

Entangling Two Qubits

Entangling two qubits with a quantum bus

- “Dispersive coupling”: coplanar waveguide resonator detuned from either qubit frequency



Graphic: Chow PhD thesis

Generating a CNOT gate

$$CNOT = -\frac{1}{\sqrt{2}}\left(\overbrace{\sigma_0 \otimes \sigma_x}^{\text{single qubit gate}} + i\sigma_0 \otimes \sigma_0\right)\frac{1}{\sqrt{2}}\left(\sigma_0 \otimes \sigma_0 + i\overbrace{\sigma_z \otimes \sigma_x}^{\text{two qubit operation}}\right)\frac{1}{\sqrt{2}}\left(\overbrace{\sigma_z \otimes \sigma_0}^{\text{single qubit gate}} + i\sigma_0 \otimes \sigma_0\right)$$

$$= -\frac{1}{2\sqrt{2}}\begin{bmatrix} i & 1 & 0 & 0 \\ 1 & i & 0 & 0 \\ 0 & 0 & i & 1 \\ 0 & 0 & 1 & i \end{bmatrix}\begin{bmatrix} 1 & i & 0 & 0 \\ i & 1 & 0 & 0 \\ 0 & 0 & 1 & -i \\ 0 & 0 & -i & 1 \end{bmatrix}\begin{bmatrix} 1+i & 0 & 0 & 0 \\ 0 & 1+i & 0 & 0 \\ 0 & 0 & -1+i & 0 \\ 0 & 0 & 0 & -1+i \end{bmatrix}$$

$$= \frac{1-i}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- We can make a CNOT with two single qubit operations (X and Z) along with a single two qubit operation ZX

Rigetti: PhD Thesis

Hamiltonian for two coupled qubits

- Hamiltonian for two coupled qubits, each with an RF drive perturbation

$$H = \frac{1}{2}\hbar\omega_1\sigma_1^z + V_1\cos\left(\omega_1^{rf}t + \phi_1\right)\sigma_1^x + \frac{1}{2}\hbar\omega_2\sigma_2^z + V_2\cos\left(\omega_2^{rf}t + \phi_2\right)\sigma_2^x + \frac{1}{2}\hbar\omega_{xx}\sigma_1^x \otimes \sigma_2^x$$

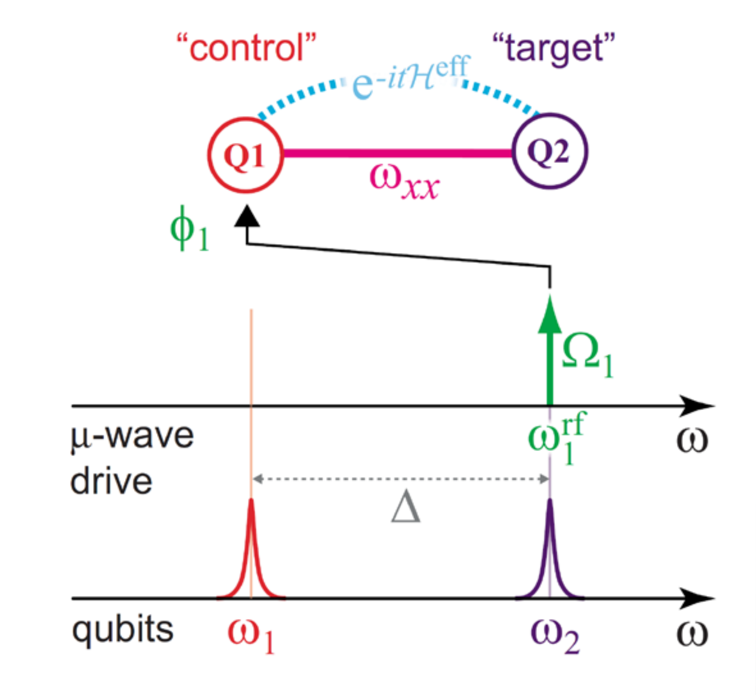
- If V_2=0, and we drive the first qubit at the resonance frequency of the second, we have the *cross-resonance condition* $\omega_1^{rf} = \omega_2$
- After multiple transformations and making the rotating wave approximation, this can be expressed

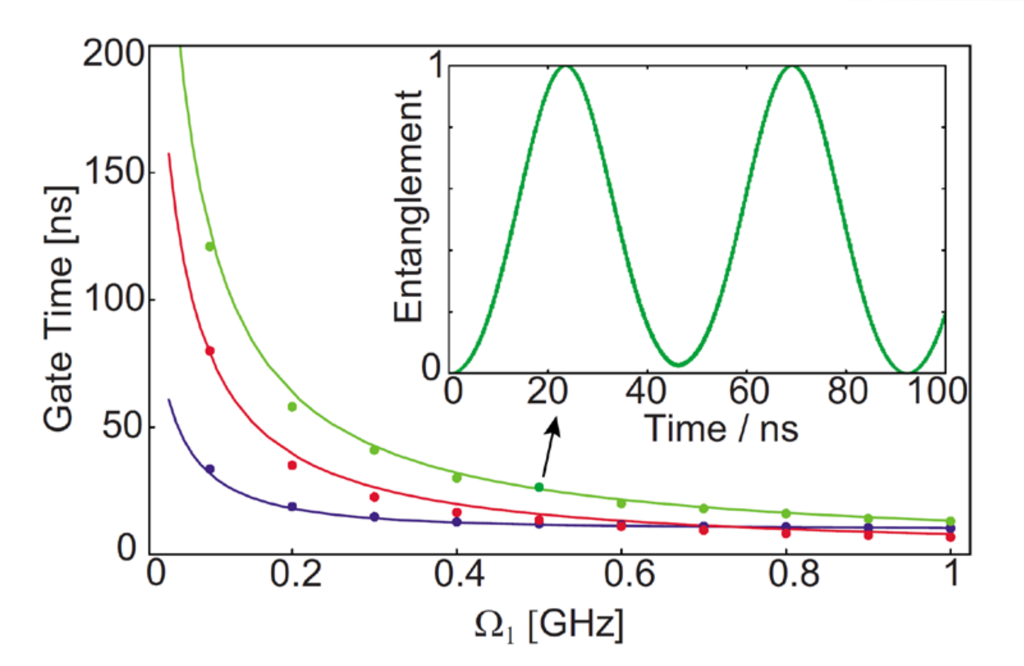
$$H^{eff} = \frac{1}{2}\hbar\omega_{xx}^{eff}\left(\cos\phi_1\sigma_1^z \otimes \sigma_2^x + \sin\phi_1\sigma_1^z \otimes \sigma_2^y\right), \quad \omega_{xx}^{eff} = \frac{V_1\omega_{xx}}{2\hbar\left(\omega_2 - \omega_1\right)}$$

- When $\omega_{xx}^{eff}t = \pi/2$ this implements a ZX gate, and along with two single qubit gates this enables the realization of a CNOT

Cross-resonant coupling

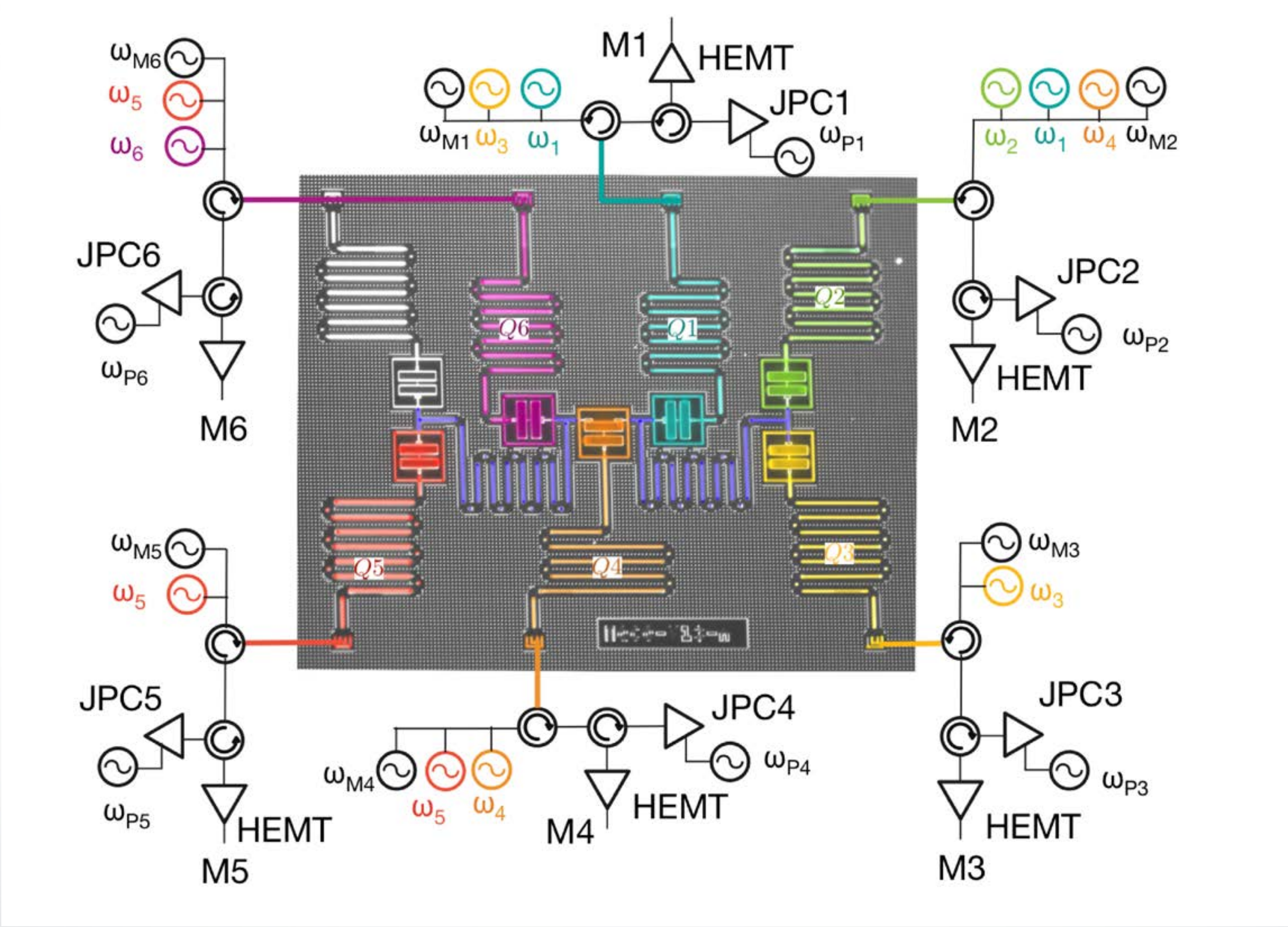
- Frequency switches on coupling
- Amplitude controls the gate speed
- Phase determines the two-qubit gate





Rigetti and Devoret

Example IBM Architecture



Kandala et al.