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Sandia National Laboratories

CASL Consortium for Advanced Simulation of LWRs

Skeptical Programming and Selective Reliability

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Skeptical Programming

- Expect rare faulty computations
- Use analysis to derive cheap "detectors" to filter large errors
- Use numerical methods that can absorb *bounded error*

Algorithm 1: GMRES algorithm

```
for l = 1 to do
  r := b − Ax^(j−1)
  q_1 := r/||r||_2
  for j = 1 to restart do
    w_0 := Aq_j
    for i = 1 to j do
      h_{i,j} := q_i · w_{i−1}
      w_i := w_{i−1} − h_{i,j} q_i
    end
    h_{j+1,j} := ||w||_2
    q_{j+1} := w/h_{j+1,j}
    Find y = min ||H_j y − ||b|| e_1||_2
    Evaluate convergence criteria
    Optionally, compute x_j = Q_j y
  end
end
```

GMRES

Theoretical Bounds on the Arnoldi Process

$$\|\mathbf{w}_0\| = \|\mathbf{A}\mathbf{q}_j\| \le \|\mathbf{A}\|_2 \|\mathbf{q}_j\|_2$$
$$\|\mathbf{w}_0\| \le \|\mathbf{A}\|_2 \le \|\mathbf{A}\|_F$$

From isometry of orthogonal projections,

$$|h_{i,j}| \le \|\mathbf{A}\|_F$$

- h_{ij} form Hessenberg Matrix
- Bound only computed once, valid for entire solve

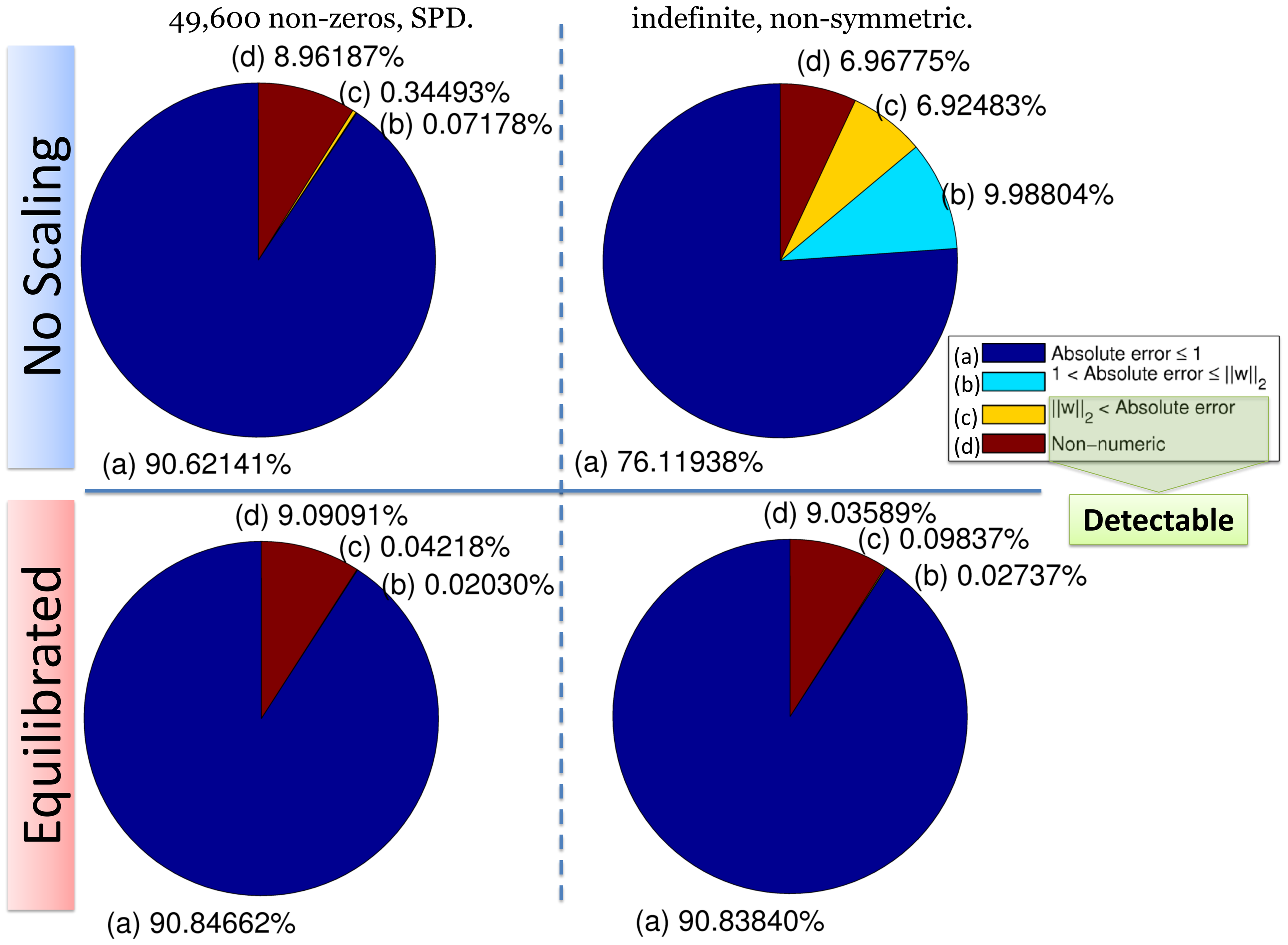
Data Representation and SDC

IEEE-754 floating point numbers: biased exponents force numbers less than 1.0 to have binary patterns that will produce absolute error less than 1.0 for majority of bits when computing dot products.

Count of possible bit flip perturbations for all Arnoldi iterations

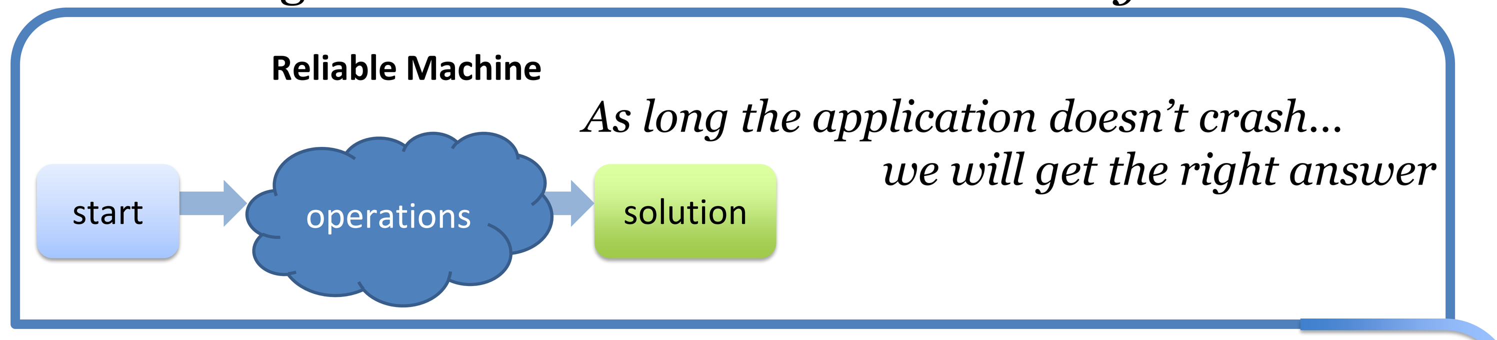
Matlab gallery matrix for the **Poisson equation**, 49,600 non-zeros, SPD.

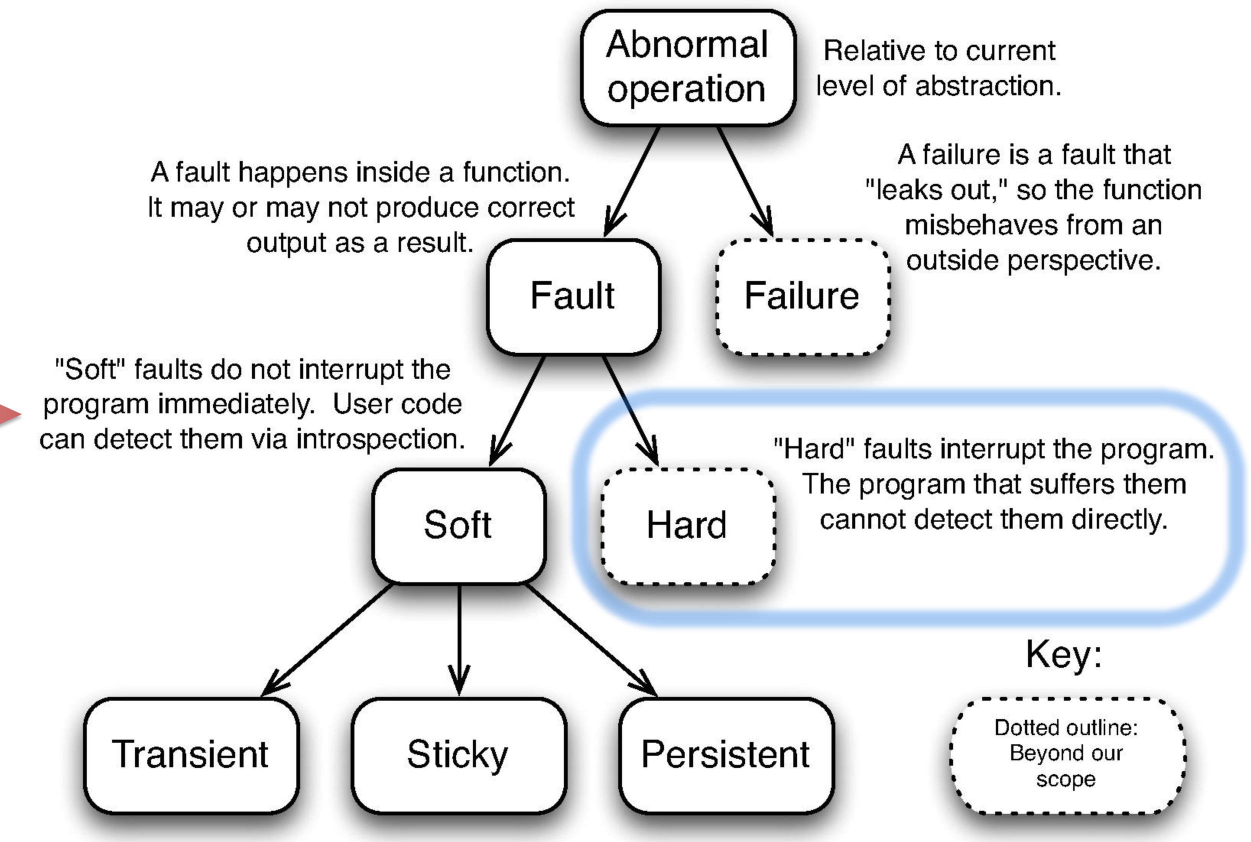
CoupCon3D matrix from Sparse Suite, 17.5 million non-zeros, indefinite, non-symmetric.



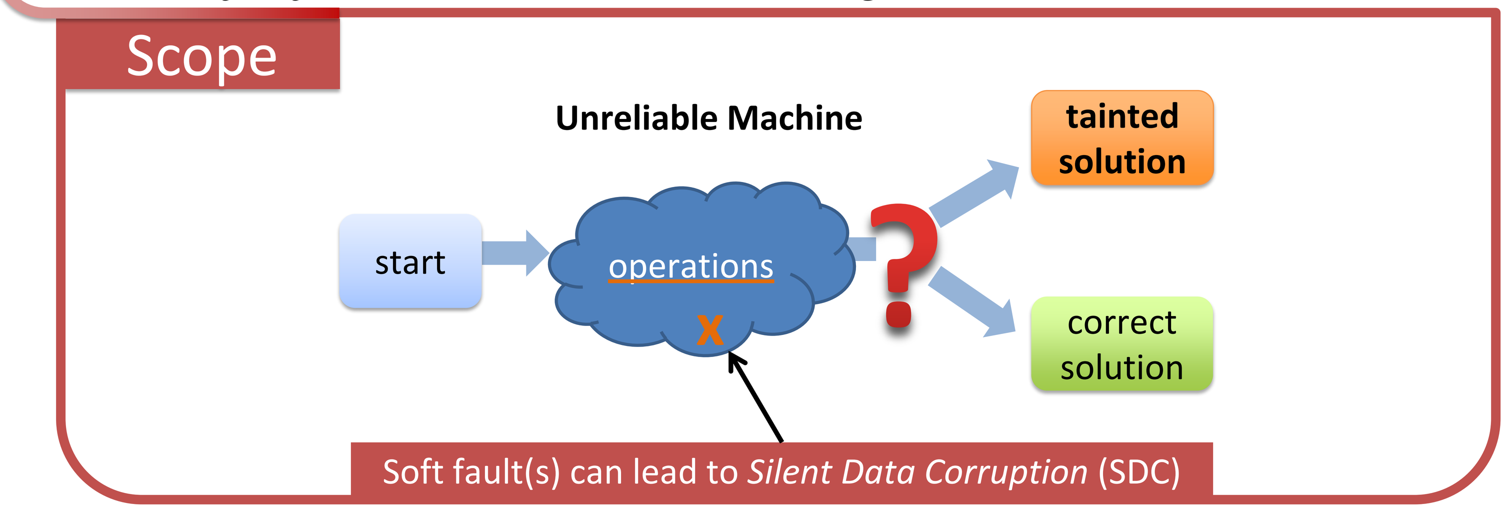
Myth of a Reliable Machine

Current algorithms assume *numerical reliability*



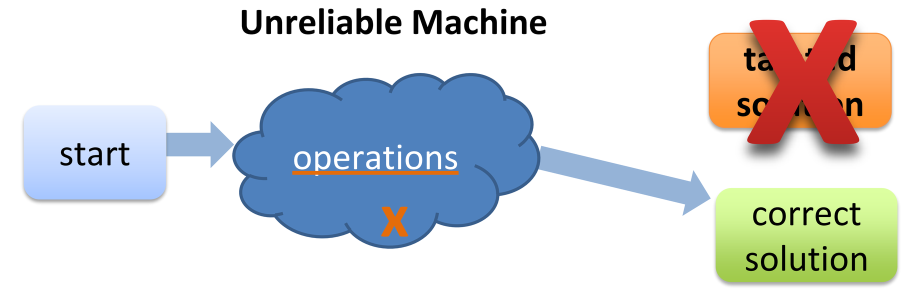


What if a fault does not crash the system, but taints our data?



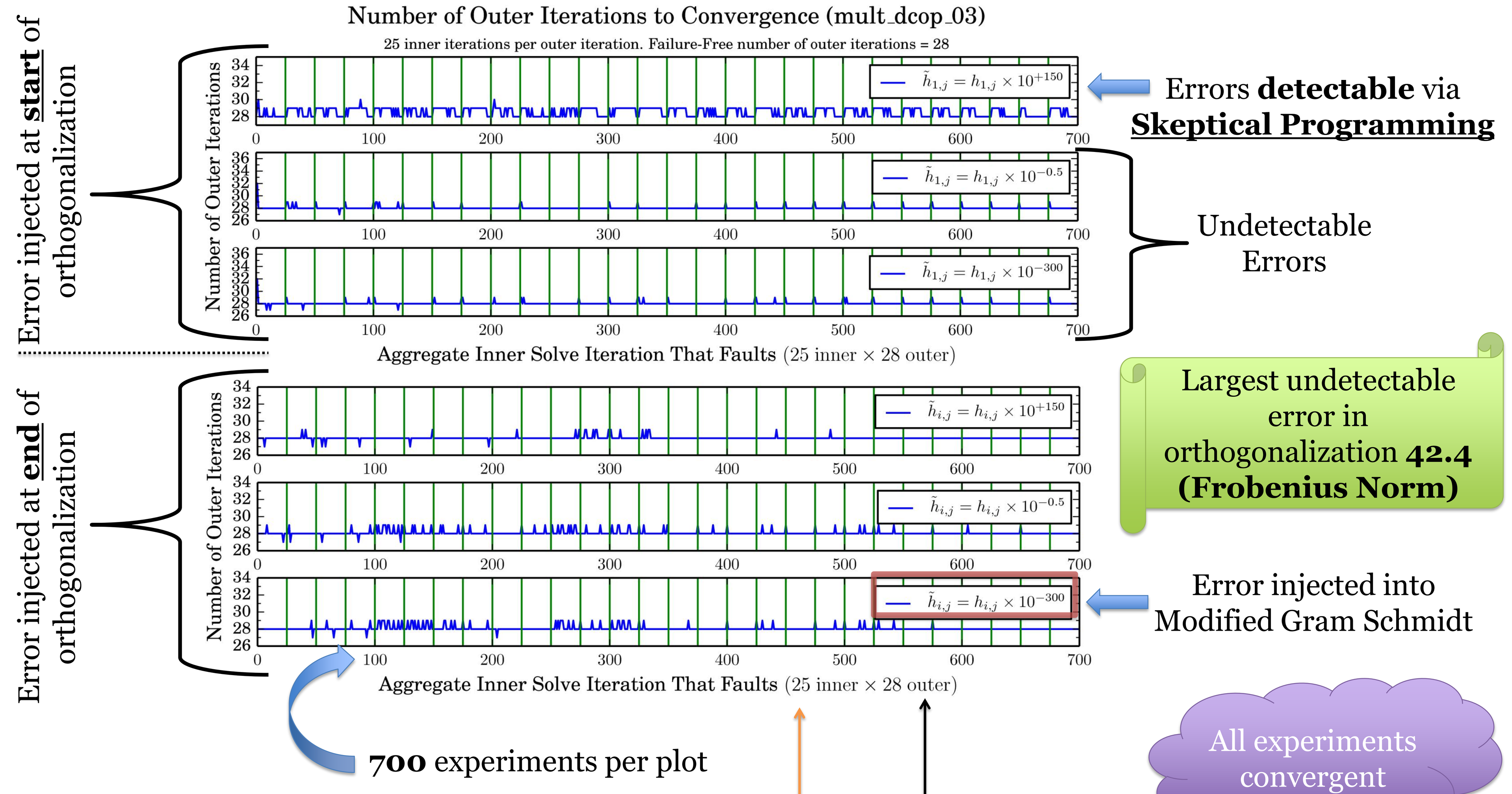
Goals

- Selective Reliability
  - *Work through* SDC or raise error if we can not
- Skeptical Programming
  - *Bound the error* from SDC when possible
- Co-design
  - Exploit *data representation* to minimize error from SDC
  - Runtime system aware of reliable vs. unreliable computation



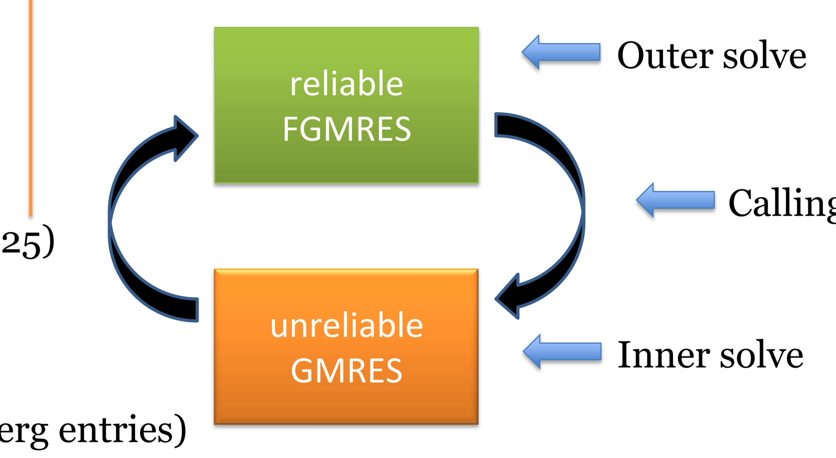
Skeptical Programming + Selective Reliability

Non-Symmetric System



FT-GMRES[2]

- Flexible GMRES + GMRES (as preconditioner)
- Selective Reliability
  - Reliable outer solver (Flexible GMRES)
  - Unreliable inner solver (GMRES)
  - *Realization of Sandbox model*
- Unreliable GMRES – fixed number of iterations (25)



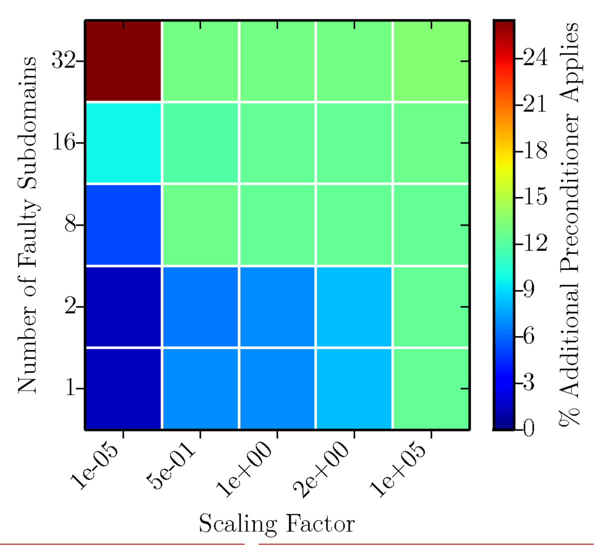
Skeptical Programming

- Used in **unreliable** GMRES
- Bound on projection coefficients (upper Hessenberg entries)
- Least squares solver hardened against nearly singular upper Hessenberg.

Opaque Preconditioners

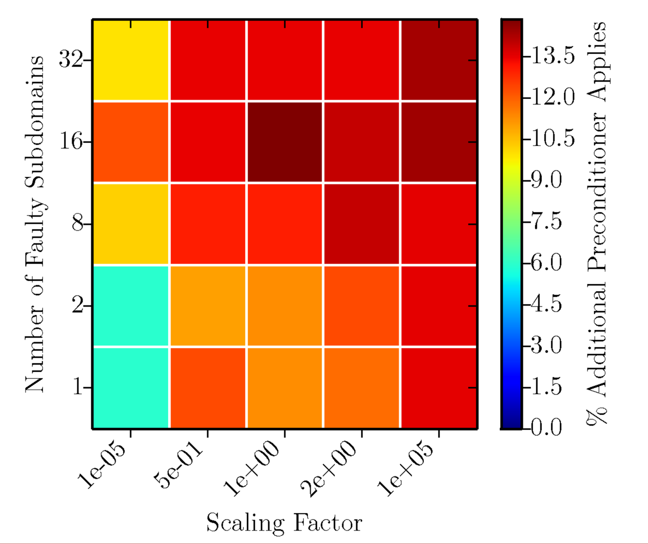
- Flexible GMRES + GMRES(25) + ILU(0)
- Color indicates the relative increase in preconditioner applications
- 32 subdomains, y-axis indicates how many subdomains were faulty (single fault injected)
- Faults modeled to be bad enough to cause plain GMRES to diverge
  - Faulty subdomain permutes output of ILU
  - X-axis, faulty subdomain optionally scales permuted values

Average



Key Findings: As long as all subdomains are not faulty, small errors always produce lowest overhead.

Standard Deviation



Acknowledgments

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References

1. "Quantifying the Impact of Single Bit Flips on Floating Point Arithmetic" by J. Elliott, F. Mueller, M. Stoyanov, C. Webster", invited talk at SIAM Conference on Computational Science and Engineering, Feb 2013, see TR 2013-2, Dept. of Computer Science, North Carolina State University, Mar 2013.
2. P. G. Bridges, K. B. Ferreira, M. A. Heroux, and M. Hoemmen. Fault-tolerant linear solvers via selective reliability. ArXiv e-prints, 2012.

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