Prioritized Token-Based Mutual Exclusion for Distributed Systems

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Abstract

A number of solutions have been proposed for the problem of mutual exclusion in distributed systems. Some of these approaches have since been extended to a prioritized environment suitable for real-time applications but impose a higher message passing overhead than our approach. We present a new protocol for prioritized mutual exclusion in a distributed environment. Our approach uses a token-based model working on a logical tree structure, which is dynamically modified. In addition, we utilize a set of local queues whose union would resemble a single global queue. The prioritized algorithm has an average overhead of $O(log(n))$ messages per request for mutual exclusion with a worst-case overhead of $O(n)$, where n represents the number of nodes in the system. Thus, our prioritized algorithm matches the message complexity of the best non-prioritized algorithms while previous prioritized algorithms have a higher message complexity, to our knowledge. Furthermore, the concept of local queues can be incorporated into arbitrary token-based protocols with or without priority support to reduce the amount of messages.

1. Introduction

Common resources in a distributed environment may require that they are used in mutual exclusion. This problem is similar to mutual exclusion in a shared-memory environment. However, while the shared memory architectures generally provide atomic instructions (*e.g.,* test-and-set) that can be exploited to provide mutual exclusion, such provisions do not exist in a distributed environment. Furthermore, commonly known mutual exclusion algorithms for shared-memory environments that do not rely on hardware support still require access to shared variables.

In a distributed environment, mutual exclusion has to be provided via a series of messages passed between nodes that are interested in a certain resource. Several algorithms to solve mutual exclusion for distributed systems have been developed [2]. They can be distinguished by their approaches as token-based and non-token-based. The former ones may be based on broadcast protocols or they may use logical structures with point-to-point communication.

We introduce a new algorithm to provide mutual exclusion in a distributed environment that supports priority queuing. The algorithm uses a token-passing approach with point-to-point communication along a logical structure. Each node keeps a local queue and records the time of requests locally. These queues form a virtual global queue ordered by priority and FIFO within each priority level with regard to the property of relative fairness introduced in this paper. We neither use a global logical clock nor timestamps to avoid the associated message-passing overhead. The main purpose of this algorithm is to provide an efficient solution to the problem of mutual exclusion in distributed systems with known average and worst-case behavior and to support strict priority scheduling for real-time systems at the same time. Thus, the worst-case behavior of the algorithm can be determined for known bounds on message delays.

In the following, the algorithm is derived from a non-prioritized algorithm that uses a single distributed queue [10]. We explain why this algorithm is not easily extended to handle priorities. Instead, we develop our algorithm step-by-step by introducing priority support, local queues and token forwarding. Then, we informally show the correctness of the algorithm.

2. The Model

First, we assume a fully connected network (complete graph). Network topologies that are not fully connected can still use our protocol but will have to pay additional overhead when messages from A to B have to be relayed via intermediate nodes. Second, we assume reliable message passing (no loss, duplications, or modifications of messages) and ordered message delivery with respect to a pair of nodes, *i.e.* if two messages are sent from node A to node B, then they arrive at B in the order that they were sent from A. We do not impose any ordering of messages originating from different nodes. Our assumption is that local operations are several orders of a magnitude faster than message delivery since this is the case in today's networks and

the gap between processor speed and network speed still seems to widen. Thus, our main concern is to reduce the amount of messages at the expense of local data structures and local operations. For the property of *relative fairness* for priority request (introduced in Definition 1), we also assume bound message delays, *i.e.* deterministic latency and throughput characteristics of the communication medium.

3. The Unprioritized Algorithm

The basic idea behind our token-passing protocol for distributed mutual exclusion stems from an algorithm by Naimi *et. al.* [10]. In this decentralized algorithm, nodes form a logical tree pointing via *probable owners* towards the root. Consider Figure 1. The root T holds the token for mutual exclusion. A request for mutual exclusion travels along the probable owners (solid arcs) to the token holder. In the example, the request by A is send to B. Node B forwards the request along the chain of probable owners to T and sets its probable owner to A. When a request arrives at the current token holder, the probable owner and a *next* pointer are set to the requester, *i.e.* T sets the *next* pointer (dotted arc) and the probable owner to A. The next request from C is first sent to B. Node B forwards the request to A following the probable owners and sets its probable owner to C. Node A sets its *next* pointer and probable owner to C. When the token holder returns from its critical section, the token is sent to the target node that *next* points to. In the example, T passes the token to A and deletes the *next* pointer.

Consider the process of request forwarding again. When a request passes intermediate nodes, the probable owner is set to the requesting node. This causes a transformation of the tree to a forest. The traversed nodes form a tree rooted in the requesting node, which will be the token holder of the future. Nodes that have not been traversed are still part of the original tree rooted in the current token holder. For example, when B forwarded A's request to T, B sets the probable owner to A. At this point, one logical tree $C \rightarrow B \rightarrow A$ is rooted in A while another tree $D \rightarrow T$ is still rooted in T. Once the request reaches T, the separate trees are merged again (2nd tree of Figure 1).

Should k requests be in transit, then there may be up to $k+1$ separate trees in the forest. Once all requests have arrived at current or future token holders, the forest collapses to a single tree again rooted in the last requester. The *next* pointers form a distributed queue of pending requests from the current token holder to the last requester. A new request would simply be appended at the end as described above.

This algorithm has an average message overhead of $O(log(n))$ since requests are propagated through a tree. In the worst case, requests can be forwarded via $n-1$ messages and one additional message is needed to send the token. However, the model assumes that there is only one requester per node at a time and requests are ordered FIFO without priority support. In a multithreaded environment, multiple requests may be issued from a node. The algorithm could be easily extended to support a list of *next* pointers per node, where each pointer represents a request of a different thread. However, priority queuing cannot be supported easily as discussed in the next section.

4. Priority Queuing Support

In a prioritized environment, requests should be ordered first by priority and then (within each priority level) FIFO. Assume that a request by node N is issued at priority $\Im(N)$. The basic idea for priority support is to accumulate priority information on intermediate nodes during request forwarding. Thus, if the request by N passes along a node I, the probable owner for requests with priority $\leq \Im(N)$ is set to N. (Higher priorities denote more important requests.)

The example in Figure 2 depicts a sequence of token requests of A, B, C, and D with priority 1, 2, 1, and 3, respectively. (Disregard the local queues of nodes for now.) The token resides in node T while the requests arrive. For the sake of simplicity, we assume that the initial logical structure is a tree whose edges point to the token owner for all priorities (labeled as "top"). Each request results in new forwarding information for certain priority levels, as depicted by the labeled edges. For example, request $(A, 1)$ results in edges $A \rightarrow A$ labeled ≤ 1, which indicates that future requests from A with priority ≤ 1 are handled by A. Other edges include $A \rightarrow B(> 1)$, $B \rightarrow T(> 1)$ and $B \rightarrow A(\leq 1)$.

If we follow the unprioritized algorithm with the addition of priority constraints on edges, we had to set *next* pointers for each request. However, a *next* pointer may have to be modified when higher priority requests arrive before lower ones have been served. This indicates that the unprioritized algorithm is unsuitable to support priority queuing. In essence, it is not possible to insert new requests at arbitrary places within the distributed queue due to race conditions. If a new entry had to be added at the head of the queue and the request arrives at the current token holder, the *next* pointer of the requester R has to be set to the *next* pointer of the current token holder. Yet, while the request was in transit, other nodes may have already registered more requests with R, thereby utilizing its *next* pointer. Thus, there is a race between setting R's *next* pointer. This deficiency can be solved by local queues.

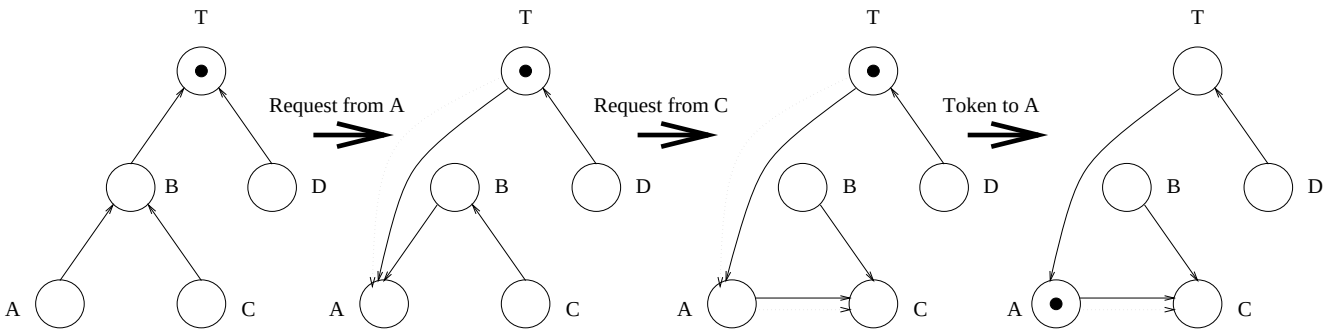


Figure 1: Unprioritized Example

5. Local Queues

The unprioritized algorithm uses a distributed queue. We can replace this queue (*i.e.*, the *next* pointers) by a set of local queues. When a node R issues a request at priority $\Im(R)$, this request propagates along the chain of probable owners until it reaches a node I with a pending request at priority $\geq \Im(R)$. It is then locally queued at I. The basic idea is that node I has to be served before our request. Thus, R's request may be stored locally until the token arrives. Once the token arrives, R's request should be served after I's if no other requests have arrived since then. This is also depicted in Figure 2, where local queues are associated with nodes T and B. Node B stores C's request at priority one since B has an outstanding request at priority two that will be served before.

A subtable problem with a set of distributed queues is the handling of FIFO queuing at the same priority level. Consider T sending the token to I. Node T has an entry for node I at the head of its local queue and possibly more entries at the same or at a lower priority. When I receives the token, the queue of T is piggybacked. Node I then has to merge its local queue and T's queue. However, if two entries have the same priority in different queues, which entry should be served first?

In a distributed system, FIFO policies can be enforced by a global logical clock via timestamps [6, 3]. However, timestamps result in additional message-passing overhead for communication with processes that never request a certain lock and overhead for acknowledgments. Instead, we utilize *local time* facilities to enforce FIFO ordering. Notice that local time only has a meaning on the current node. However, when the token is transmitted from T to I and T's queue is piggybacked, then the first request in T's queue is a request by node I. If all requests by I log the local initiation time, we are able to measure the time between request initiation $t_I(req)$ and token reception on the local node (now). This interval is called the request latency $t_I(latency)$ (see Figure 4). The latency itself is composed of the request transit time $t_I(req_transit)$, the token transit time $t_I(token_transit)$, and the token usage time $t_I(usage)$.

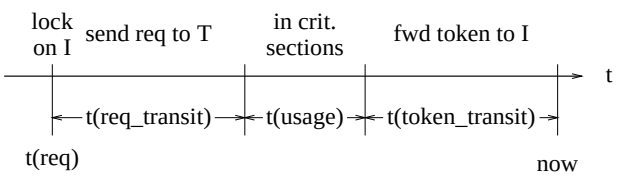


Figure 4: Events and Times for Token Requests

$$t_I(latency) = t_I(req_transit) + t_I(token_transit) + t_I(usage)$$

Index I may be replaced by other indices for other nodes. Notice that the queue piggybacked with a token message contains local requests *and* the accumulated token usage time. The token usage time can be calculated as the time spent in critical sections on nodes holding the token since request reception. Thus, latency and usage can be measured but the transit times may only be inferred indirectly due to a lack of global clocks. We can simply transform the equation to obtain the transit time:

$$\begin{aligned} t_I(transit) &= \\ & t_I(req_transit) + t_I(token_transit) = \\ & t_I(latency) - t_I(usage) = \\ & (now - t_I(req)) - t_I(usage) \end{aligned}$$

We can now determine the approximate usage times for nodes X within the local queue due to the following observation. The local queue contains requests from the current node and possibly other nodes with the associated reception time $t_X(receipt)$ of each request by the local node. Thus, the usage time for local requests from X is determined as

$$\begin{aligned} t_X(new_usage) &= \\ & now - (t_X(receipt) - t_I(transit)) \end{aligned}$$

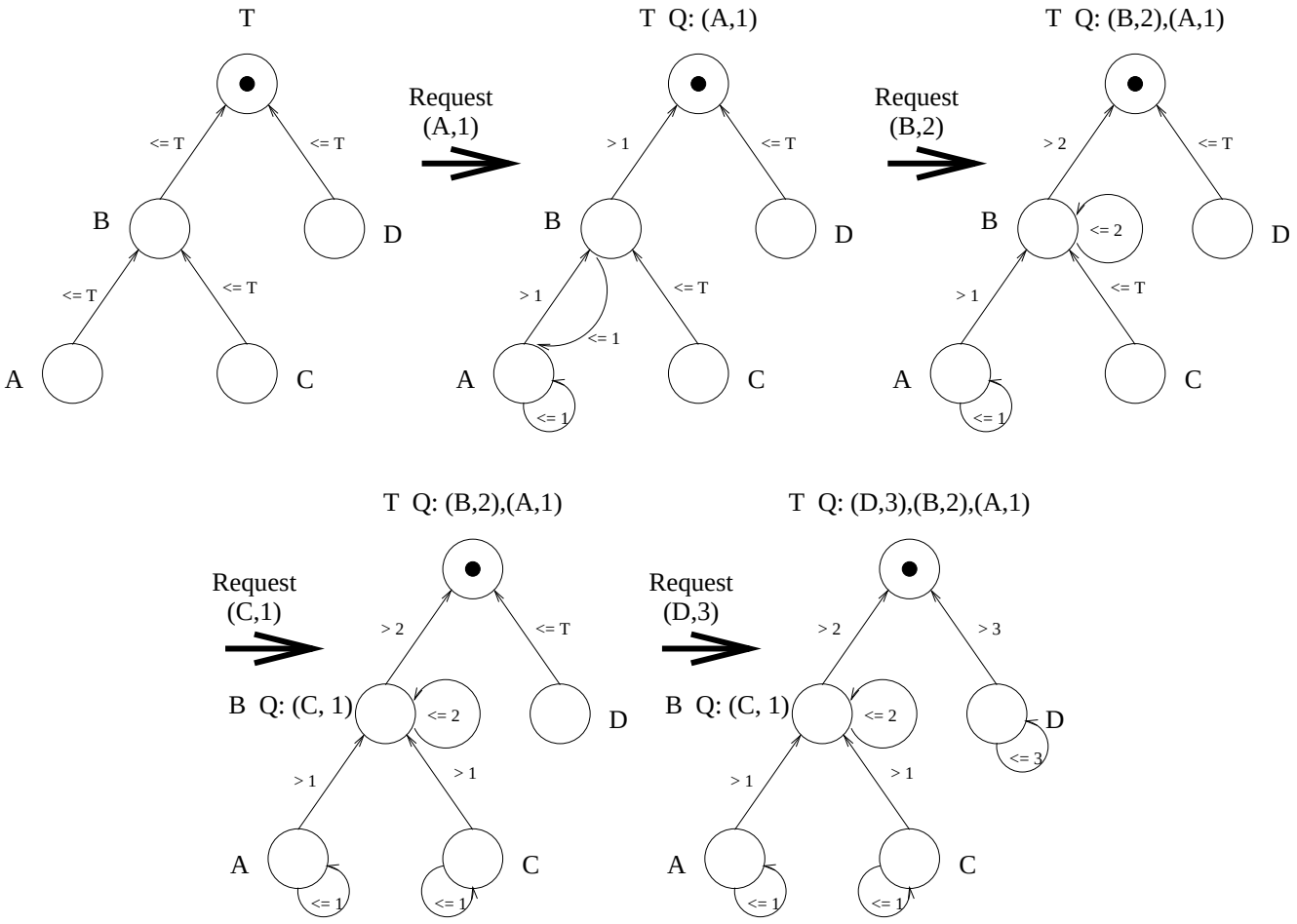


Figure 2: Example of Token Requests

Once the usage times have been determined for local requests, the local queue can be merged with the piggybacked queue from T, which already contains accumulated usage times. Requests are ordered by descending priority and, within each priority level, by descending usage time. Thus, the usage time provides a means of aging that guarantees requests to be eventually served. This avoids starvation of requests, discussed in more detail in section 9.3. The reception time of all requests is set to the current local time after merging.

The token may be used to grant access in mutual exclusion on the local node. During token usage, other requests may be received, which are then logged with a zero usage time and the current local time as a reception time. These requests are merged into the existing queue using the same precedence rules as before (descending priority and descending usage time within each priority) and, in addition, with descending reception time (within the same priority level and the same usage times).

6. Token Forwarding

Once the token is no longer being used on the local node, it may be forwarded to the first requester within the queue. The queue is also piggybacked with the forwarded token but usage times are recalculated before the forwarding to include the local usage time:

$$t_X(new_usage) = \\ t_X(usage) + now - t_X(receipt)$$

The example of Figure 2 is continued in Figure 3. The token is forwarded to nodes D and B. The previous token holder retains only one link to the next token holder for requests at any priority level. Notice that the figure does not depict usage times and their adjustment upon token forwarding.

Typically, a request from node R propagates along intermediate nodes I before reaching the token holder T, as depicted in Figure 5a. Token forwarding may, however, result in a race condition that can be characterized as follows. A request from a node R may pass

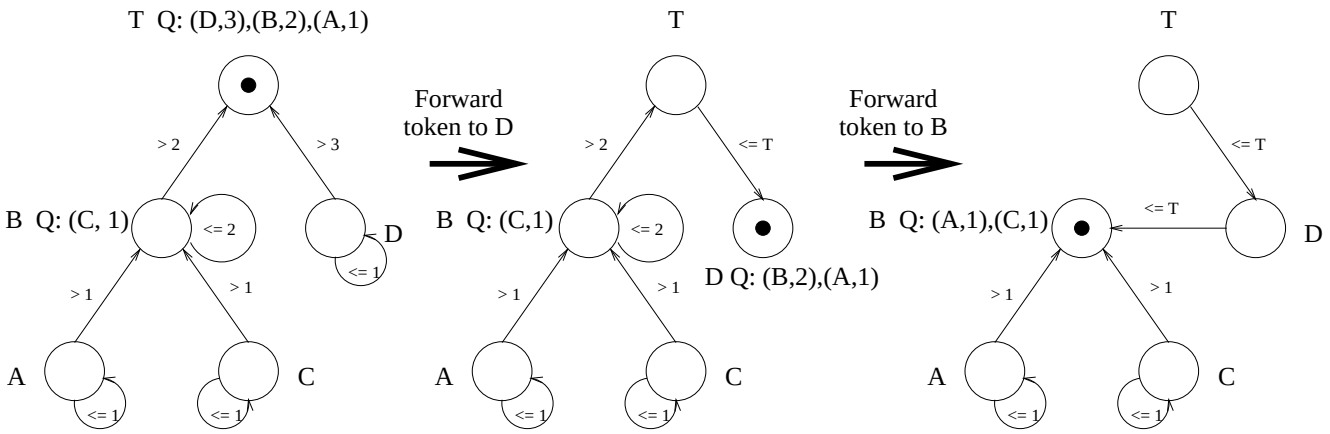
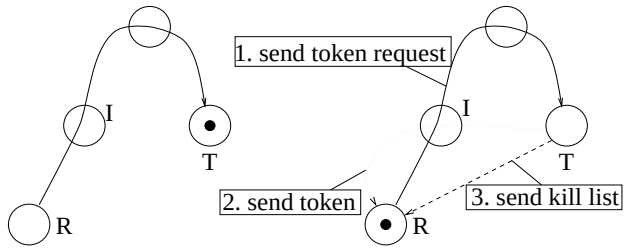


Figure 3: Example of Token Forwarding

intermediate node I, where another lower priority request had already been issued. If the token is sent from node T while request R is in transit and has already passed node I, a race between the request and the token location exists. For example, in Figure 3 consider a request $(A, 4)$ being issued and passed *via* B and T. Meanwhile, T has already forwarded the token to D and D has forwarded it to B. Hence, the new request would trail behind to D and B, the latter being a node the request had passed once before (forming a cycle).



(a) Request Reaches Token (b) Request Trails Token

Figure 5: Abstract Model of Token Forwarding

Returning to the abstract characterization of the race condition, the communication overhead may no longer be bound by the number of nodes in the system since the request from R may pass repeatedly by node I. We can avoid this situation by adding request R to the local queue of all intermediate nodes I. Should the token then be passed to I before request R reaches T (but after passing by I), node I would forward the token to the higher priority requester R before using it, as depicted in Figure 5b. Request R logs all intermediate nodes on the way and stops propagating when an edge along the probable owner path leads to an already logged node. Thus, request R stops propagating at T since T's probable owner is I (T sent the token to I) and the request had already passed I. In addition, a kill list of entries added to local queues on intermediate nodes is passed to the token requester R. Node R ensures that the request for mutual exclusion is only granted after this kill list has been received.

The added request on intermediate nodes will be processed when the token reaches the intermediate node the next time. There are two subcases. Either the token passes by intermediate node X before reaching R, for example node I in Figure 5b. Then, the request will be merged in with other pending requests and it is locally recorded that the request has been served.

In the other case, the token passes by intermediate node X after reaching R. A kill list indicates the entries to be removed from local queues of all intermediates nodes. This list is received before or while R holds the token (indicated by the dashed edge in Figure 5b). When the token reaches a node in the kill list, entries of the local queue are removed if the kill list indicates this.

7. Message Overhead

In an environment with n nodes, the resulting algorithm requires an average of $O(log(n))$ messages for a request for mutual exclusion since it uses a logical tree similar to the algorithm by Naimi *at. el.* [10]. In the worst case, the links between nodes form a chain such that $n - 1$ messages are required for the propagation of the request. This should be obvious from the last section, where we ensured that token requests cannot propagate in cycles, in other words, they cannot pass a node twice. There are at most two more messages required to send the token and one message to send the kill list (see Figure 5b).

Our algorithm requires more overhead in terms of local bookkeeping than the algorithm by Naimi *et. al.*.

In addition, we have to piggyback data structures with requests and token sends. Nonetheless, if the amount of messages is the main cause of overhead, as we assumed, our algorithm is the most efficient implementation of distributed mutual exclusion with priorities that we know of.

8. The Prioritized Algorithm

The prioritized algorithm is depicted in Figure 6. A monitor is used to abstract from local locks required on the current node due to concurrent activations of the service routines. A condition variable `cond` is used to provide conditioned synchronization locally when waiting for the token. The routines `lock` and `unlock` are the user interface for distributed mutual exclusion. A token sent in `forward_token` is received in `receive_token`. A request is initially issued in `lock` but may also be issued in `receive_token` to provide forwarding. A request is received in `receive_token_request`, and is either forwarded or, if next probable owner has already been visited, the kill list is sent. The kill list is received in `receive_kill_list`.

9. Informal Proof

We will give an informal outline of the proof by first showing that mutual exclusion is satisfied, then that deadlocks cannot occur and finally that requests cannot starve in the absence of higher priority requests.

9.1. Mutual Exclusion

In the initial state, only one node, the start node, has the token. The token may be passed during an `unlock`, a `receive_token`, or a `receive_token_request`. In all cases, the token is passed only to one node and the current node loses it. A process may only enter a critical section when the node holds the token. Thus, mutual exclusion is guaranteed between nodes. Within a node, local tasks (threads) may compete for a critical section but the monitor semantics ensure than only one task can enter the critical section since a task may proceed only past the `ENDIF` in `lock` if `token_available` is true, and will then set the flag to false before entering the critical section (and exiting the monitor).

9.2. Absence of Deadlock

In the following, we restrict ourselves to token requests from nodes other than the token holder. We have to ensure that if the token is available, a request will eventually be known to the token holder. Once the token holder knows of at least one request, the token is forwarded to the first request in the local queue by invoking `forward_token`. Finally, circular dependencies due to token forwarding must not occur. The property that a request becomes known to the token holder is shown by the following three lemmas.

Lemma: For a requesting source node R (other than the token holder) with priority $\Im(R)$, there is a chain of owners leading either to the token holder or to another requester with priority $\geq \Im(R)$.

Proof: Initially, all nodes without the token point to the token holder T via their owner sets. Assume that the lemma holds at some point. If R issues a request at $\Im(R)$ and $R \neq T$, then the request will propagate either to T or to an intermediate node I with outstanding request at priority $\geq \Im(R)$. Node R and all nodes N between R and T (or I) forward the request to a next node in the chain. In addition, they (and nodes I or T) add the entry $(R, \Im(R))$ to its owners. Thus, all nodes along the traversed path form a chain leading to node R at priority $\Im(R)$. A node X that was not traversed may either point (directly or indirectly) into the middle of this new chain, in which case its path also leads to R for $\Im(R)$. Or, alternatively, node X never leads to this new chain. Then, the old chain is unmodified and still points to T. If T had already received R's request (or another request from node S) and sent the token to R (or S), the chain now extends further from T to R (or S), the new token holder. On the other hand, if T still holds the token (and R's request is in transit), then the chain still ends in T, the token holder.

Lemma: For request R with priority $\Im(R)$, if a chain of owners from R to I does not lead to the token owner T, there is at least one node N on this chain (R and I inclusive) that either has a request in transit toward T or N is in T's local queue of requests. Thus, we restrict our discussion to requests at the same priority level from here on.

Proof: This follows immediately from the last proof and the fact that a request is put into the queue in `receive_token_request` if the target's owners are empty or point to self for this request, which is the case for T.

Lemma: If request R is in transit to the token holder T, it will be queued with the current or a future token holder or an intermediate node N that the request traversed.

Proof: If there is a path from R to T and T remains the token holder, then R's request reaches T eventually and is added to the local queue. If the token is passed on from T, the path will extend from T to the new token holder; R's requests will keep propagating until is reaches the new token holder or until the path leads to a node I that request R has already traversed. In the latter case, the request stops propagating before I (due to the `visited_set`), I is the token holder and R'a request is present on I's local queue.

Thus, we showed that a request will eventually be

```
MONITOR distributed_mutex IS
  VAR owners,   /* set of (node, prio) tuples of probable owners, sorted by prio*/
      queue,    /* set of (node, prio, rect, uset, reqt) in local Q,           */
                /* 1) decr. prio 2) decr. uset 3) decr. rect                   */
      requests, /* set of (node, prio, t) of local requests, sorted as queue   */
      kill_list,/* set of (node, node, prio, reqt) of future deletions, unsorted*/
      req_done, /* set of (node, reqt) of requests whose kill list is received */
      req_handled,/*set of(node, reqt) of requests whose kill list is handled  */
      critical_section, /* tuple (node, prio, t), set if local crit. sect. used */
      token_available,  /* flag, set if token on local node and available      */
      cond;     /* condition for monitor, signalled if token locally available */
      kill_cond;/* condition for monitor, signalled if new kill list available */

  PROCEDURE init IS
    owners = (start, top);
    queue = requests = kill_list = NULL;
    token_available = (self == start);
    IF token_available THEN
      owners = NULL;
  END init;

  PROCEDURE lock (prio) IS
    requests += (self, prio, now);
    (R, prioR) = find_last_or_return_first (owners) st. prioR >= prio;
    /* find last entry in owners satisfying the condition or return 1st entry */
    owners -= all (X, prioX) st. prioX <= prio;
    owners += (self, prio);
    IF !token_available THEN
      kill = (self, self, prio, now);
      queue += (self, prio, now, 0, now);
      send token request (self, prio, now, kill, self) to R;
      WHILE (!token_available)
        wait(cond);
      queue = tail(queue);
    ENDIF;
    token_available = FALSE;
    critical_section = head(requests);
    requests = tail(requests);
    owners -= all (X, prioX) st. prioX > prio;
  END lock;

  PROCEDURE unlock IS
    (self, prio, t) = critical_section;
    IF owners(prio) == self AND requests(prio) == NULL THEN
      owners -= (self, prio);
    IF queue != NULL THEN
      forward_token();
    ELSE
      token_available = TRUE;
  END unlock;

  PROCEDURE forward_token() IS
    FORALL (X, prioX, rect, uset, reqt) in queue DO
      uset += now - rect; /* accumulated usage times */
    send token(queue, kill_list) to node(head(queue));
    kill_list = NULL;
    owners = NULL;
    FOREACH (X, prioX, rect, uset, reqt) in queue DO
      owners -= (Y, prioX);
      owners += (X, prioX);
    UNTIL X == self OR end of queue OD;
```

```
PROCEDURE receive_token(global_queue, global_kill_list) IS
  FORALL (Y, X, prioX, reqt) in global_kill_list DO
    IF Y == self THEN
      queue -= (X, prioX, rect, uset, reqt); /* delete served token forwards */
      global_kill_list -= (Y, X, prioX, reqt);
      IF (X, reqt) in req_handled THEN
        req_handled -= (X, reqt);
      ELSE
        req_done += (X, reqt);
    ENDIF;
  OD;
  kill_list += global_kill_list;
  (self, prio, t_rec, t_usage, t_req) = head(global_queue);
  t_transit = now - (t_req + t_usage);
  FORALL (X, prioX, rect, uset, reqt) in queue DO
    uset += now - (rect + t_transit); /* new usage times */
  queue += global_queue; /* merge Qs, desc. prioX, desc. uset, desc. t */
  FORALL (X, prioX, rect, uset, reqt) in queue DO
    rect = now;
  (R, prioR, rect, uset, reqt) = head(queue);
  IF R == self THEN
    WHILE NOT (self, reqt) in req_done DO
      wait(kill_cond); /* wait for kill list if not received yet */
    req_done -= (self, reqt);
    kill_list -= (self, self, prioR, reqt);
    token_available = TRUE;
    signal(cond);
  ELSE
    req_handled += (R, reqt);
    forward_token();
  ENDIF;
END receive_token;

PROCEDURE receive_token_request(R, prioR, reqt, kill, visited_set) IS
  (S, prioS) = find_last_or_return_first (owners) st. prioS >= prioR;
  /* find last entry in owners satisfying the condition or return 1st entry */
  IF owners == NULL OR S == self THEN
    queue += (R, prioR, now, 0, reqt);
    IF token_available AND head(queue) == (R, prioR, rect, uset, reqt) THEN
      kill_list += kill;
      forward_token();
    ENDIF;
  ELIF S in visited_set THEN /* token forwarded to node in owners chain */
    send kill list (kill, reqt) to R;
  ELSE
    IF requests != NULL THEN /* make sure the token will come here eventually */
      kill += (self, R, prioR, reqt);
      queue += (R, prioR, now, 0, reqt);
    ENDIF;
    visited_set += self;
    send token request (R, prioR, reqt, kill, visited_set) to S;
    owners -= (X, prioR);
    owners += (R, prioR);
  ENDIF;
END receive_token_request;

PROCEDURE receive_kill_list(kill, reqt) IS
  kill_list += kill;
  req_done += (self, reqt);
  signal(kill_cond);
END receive_kill_list;
```

recognized by the token holder, even if the token holder changes while the request propagates. In addition, circular forwarding cannot occur since token forwarding can only take place if a higher priority request traverses an intermediate node that requested a lower priority request before. Since we assume a finite number of priority levels, there can only be a finite number of higher priority requests being served via token forwarding.

9.3. Absence of Starvation

Starvation in a protocol with strict priority ordering can be reduced to starvation within one priority level. This is due to the fact that higher priority requests *intentionally* bypass lower priority requests.

We have already showed that a request from R at a certain priority is queued in some local queue at node N. We will also show that the token is eventually sent to N at the same priority level. Once the token is received on N, the local queue is merged with the token queue by calculating the usage time relative to the request by N that caused the token to be forwarded to N. Thus, the initial calculation of the usage time of a request depends on the reception time of this request on the node where it is merged with the token queue. In other words, while request R is in transit to node N, other requests may still be registered before R's request. But once request R has reached N and is included in the local queue, future requests and requests in transit at the same priority are guaranteed to be served after R's request, even in the absence of global time facilities.

Assuming that there is a finite number of tasks (threads) to issue requests, there can only be a finite number of requests ahead of R's request (once it is queued at node N). The requests ahead of R may be within any local queues and the token queue and their number cannot increase once R's request is in N's local queue. The token will be forwarded to the nodes with requests ahead of R before it is finally forwarded to R. Thus, R will be served in a finite time assuming that critical sections take a finite amount of time.

9.4. Strict Priority Scheduling

A strict priority protocol requires that higher priority requests are served before lower ones, even if issued later. In a distributed environment, two events on different nodes do not have a temporal relationship in the absence of global time facilities. However, we established that the worst-case communication overhead for propagating a request is bound by $n-1$ messages. Thus, we can formulate the property of *relative fairness* with respect to the message overhead.

Theorem 1 *If two requests are issued Δ time units apart and the communication overhead for handling a request is at most k messages, bound by $\omega(k)$ time units, then the requests will be served in the same order as issued if $\Delta > \omega(k)$.*

Proof: If the first request is issued at $t(0)$ and takes k messages, it would be handled at $t(1)$ with $t(1) \leq t(0) + \omega(k)$. The second request arrives at $t(2)$ with $t(2) \geq t(0) + \Delta$. Since $\Delta > \omega(k) \Rightarrow t(0) + \Delta > t(0) + \omega(k) \Rightarrow t(2) > t(1)$.

Definition 1 *An algorithm guarantees* relative fairness *if any two requests issued Δ time units apart are served in the same order as issued.*

The above algorithm is relatively fair if $\Delta > \omega(n-1)$ since $n-1$ is the worst-case message overhead for propagating a request. Notice that "handling" a request means registering the request in this case since once a request stops propagating (being in transit), future requests (including those in transit) will be served later, as discussed in the context of starvation. The message size is bound by the transmitted queue and the kill list. The former is bound by the number of tasks (threads) in the entire system, the later is bound by the number of nodes. For a communication medium with bound message delay (deterministic latency and throughput characteristics), the overhead can be calculated based on these bounds. Thus, two requests are guaranteed to be served according to strict priority ordering if they are issued at least Δ time units apart.

10. Future Work

A formal proof for the algorithm has yet to be worked out. We are also working on a simulation environment for our algorithm. On one hand, we want to investigate the performance of the prioritized algorithm. On the other hand, we want to compare the performance of the unprioritized algorithm by Naimi *et. al.* [10] with our prioritized algorithms when all requests are issued at the same priority. After all, the concept of local queues combined with local timekeeping can be used to enhance the unprioritized algorithm as well to reduce the amount of messages. The simulation study should quantitative this claim. Notice that logging of requests on intermediate nodes, as described in section 6., is not necessary in the absence of priorities. We also intend to implement the prioritized algorithm in DSM-Threads, an environment with distributed threads. This environment is similar to POSIX threads [14] in the sense that strict priority scheduling shall be enforced and FIFO scheduling within each priority level. However, distributed threads do not share any physical memory. Instead, a distributed virtual shared memory (DSM) is supported [7, 8]. The details of DSM-Threads are beyond the scope of this paper [9].

11. Related Work

A number of algorithms exist to solve the problem of mutual exclusion in a distributed environment. Chang [2] and Johnson [5] give an overview and compare the performance of such algorithms. Goscinski [4] proposed a priority-based algorithm based on broadcast requests using a token-passing approach. Chang [1] developed extensions to various algorithms for priority handling that use broadcast messages [13, 12] or fixed logical structures with token passing [11]. Other mutual exclusion algorithms (without token passing) employ global logical clocks and timestamps [6]. These algorithms can be readily extended to transmit priorities together with timestamps. However, all of the above algorithms, except Raymond's, have a message complexity larger than $O(log(n))$ for a request. Finally, Raymond's algorithm uses a fixed logical structure while we use a dynamic one to allow the addition and deletion of nodes. Furthermore, Raymond needs $O(log(n))$ messages to send the token to a requester, where our algorithm only requires either one or three messages (at the same priority level).

12. Conclusion

The main contribution of this paper is a prioritized algorithm for distributed mutual exclusion with an average message overhead of $O(log(n))$ and a worst-case overhead of $n + 2$ messages per request, respectively. The algorithm is based on a token-passing scheme and uses local queues, which together form a global queue of pending requests. The queues are ordered by priority and then FIFO within each priority level. We give a framework to relate requests of different local queues to each other in terms of FIFO ordering by using only local time-keeping instead of a global logical clock with timestamps. Thus, local queues can be merged while preserving FIFO ordering. We utilize a set of logical trees with associated priority levels to let requests propagate either up to the token holder or to another requester with higher priority. The resulting algorithm for prioritized mutual exclusion requires forwarding of queues and local bookkeeping. It has the same message complexity as its best known non-prioritized counterparts while previous prioritized algorithms have a higher message complexity, to our knowledge. Furthermore, the concept of local queues combined with local time-keeping can be incorporated into arbitrary token-based protocols with or without priority support to reduce the amount of messages.

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