Modular Embedding of Problems onto Quantum Annealers

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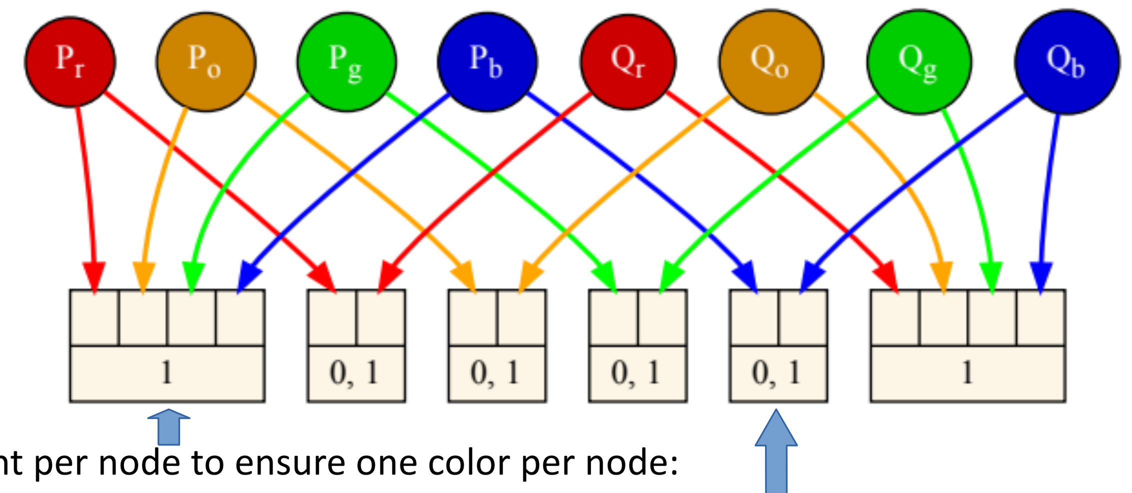
Motivation

- Quantum annealers benefit from high qubit connectivity
  - "Chains" of physical qubits used to represent single virtual qubits
- Embedding problems onto current topology is NP-Hard
- Previous work[1] shows improvement for specific problem by adding extra structural constraints
  - Separates the problem into cells of 2 qubits
  - Cells can only be embedded onto certain physical qubits on the Chimera structure
  - Similar to the idea behind DWave's *Locally Structured Embeddings*

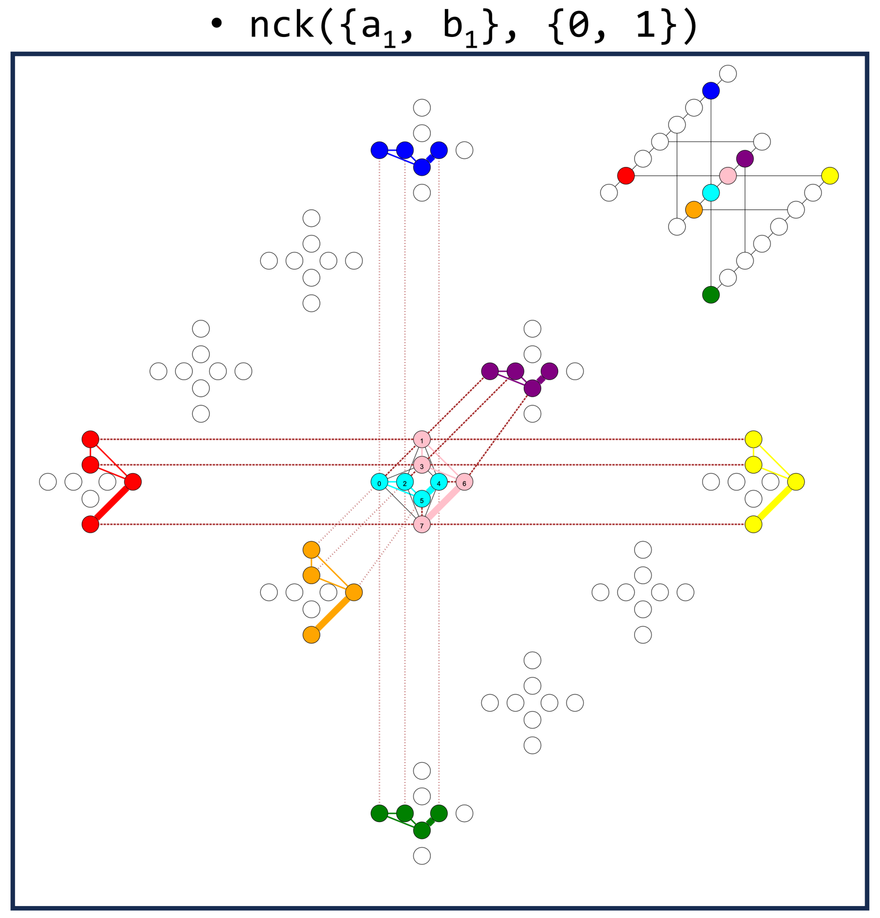
NchooseK

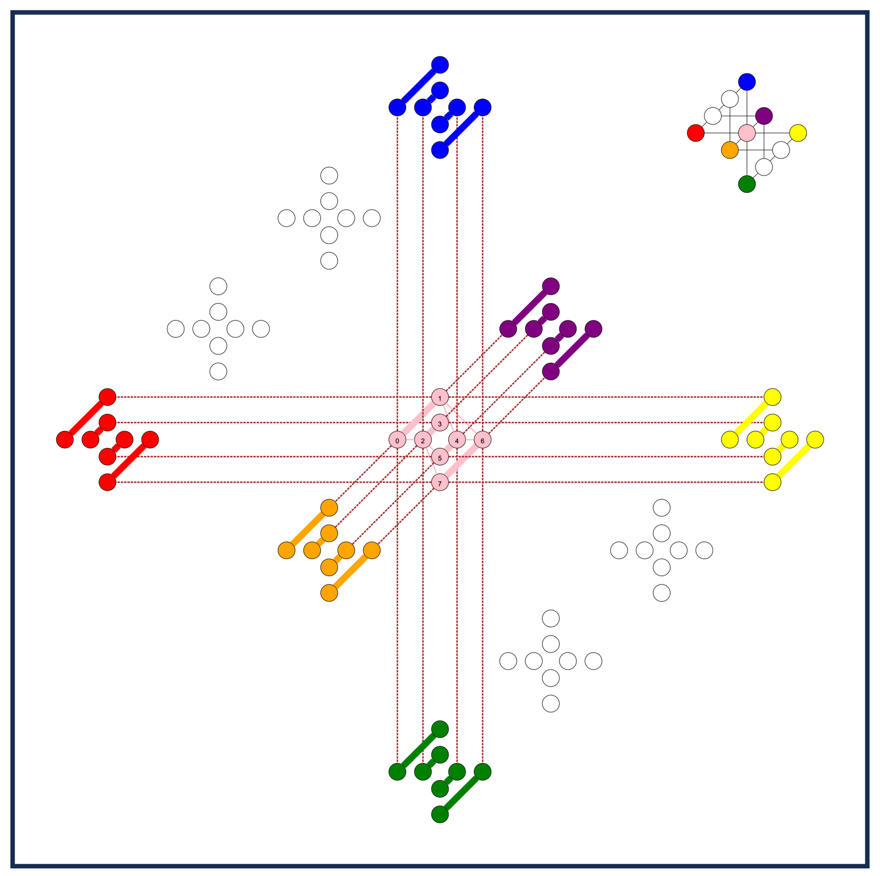
NchooseK is a domain specific, constraint-based language built for automatically setting up problems for both gate-based machines and quantum annealers[2].

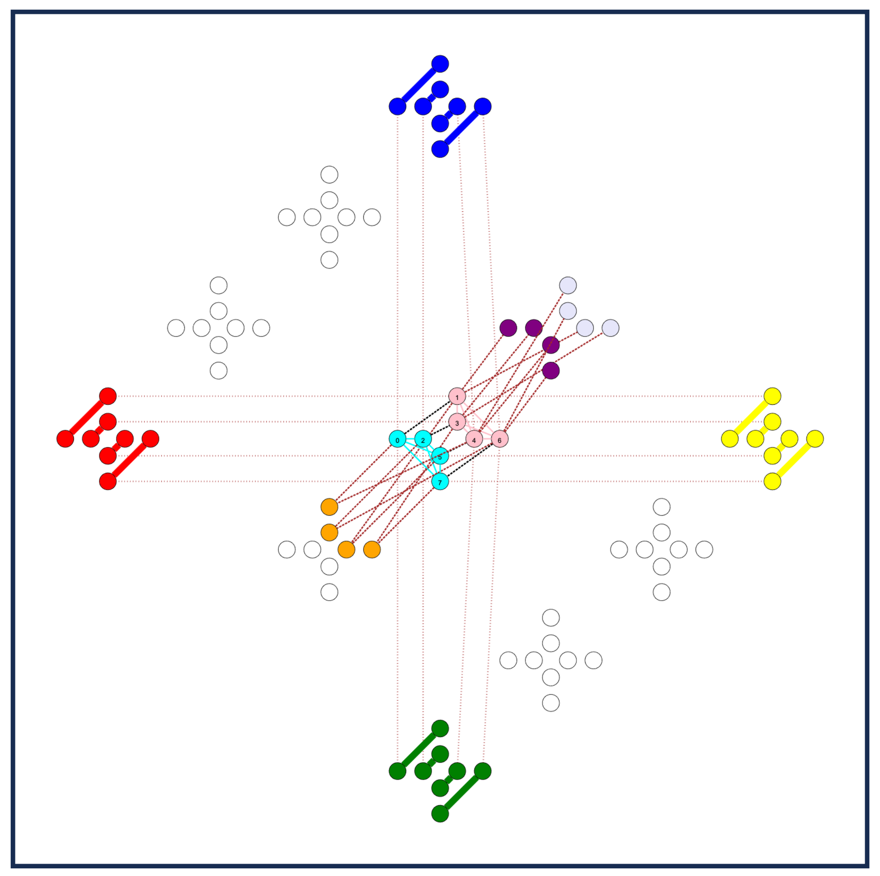
- Good candidates for modular embedding, thanks to constraint-based nature
- NchooseK uses constraints which say "Of N variables, K must be true"
- Constraints take the form `nck([N_1, …, N_n], {K_1, …, K_m})`
- Many NP problems have been solved with Nchoosek
  - One-hot encoding problems particularly suited to this type of embedding
    - Several qubits represent one variable
    - Qubit measured as |1> indicates "hot" value
  - Map coloring problem good example here
  - Map coloring uses 2 kinds of constraints, shown below:
    - Circles represent variables (regions P and Q)
    - Boxes represent constraints, number shows K for that constraint



- One constraint per node to ensure one color per node:
  - `nck({a_1, a_2, …, a_n}, {1})`
- n constraints per edge ensuring two nodes of the same color not connected:
  - `nck({a_1, b_1}, {0, 1})`







Connectivity of central cells on section of DWave Pegasus architecture
Abstract representation in upper right corner
Thick colored line indicates chain representing single virtual qubit

NchooseK Usage

```
import nchoosek
env = nchoosek.Environment()
P = ['Pr', 'Po', 'Pg', 'Pb']
Q = ['Qr', 'Qo', 'Qg', 'Qb']
for i in P + Q:
      env.register_port(i)
# One color per node
env.nck(P, {1})
env.nck(Q, {1})
# No shared color between regions
for idx, p in enumerate(P):
      env.nck([p, Q[idx]], {0, 1})
env.solve()
```

Methodology

We decided to start our investigation with the Map Color and Clique Cover problems due to their one-hot encoding lending to distinct cells with predictable connections between them.

- First tried DWave's current Pegasus architecture.
- Tried to find good encodings for maps of 3 and 4 colors
- These cells need the following properties:
- **Each cell must be a clique**
  - Each qubit within the cell connected to every other qubit within the cell
- **Corresponding qubits in connected cells must be connected to each other**
  - Ex: P_r connected to Q_r in NchooseK example
- Cell for 3 colors use 4 qubits (bottom left)
  - Thicker line indicates a chain, representing one virtual qubit
  - Cells have a degree of 4
- Two cells found for 4 colors
  - One version uses 8 qubits per cell (bottom center)
    - Cells have a degree of 6
  - Other version uses 4 qubits per cell (bottom right, unused)
    - Cells have degree of 2 or 3
    - Additional constraints on placement depending on connectivity
    - Cells need to be combined to connect with cells not on the same diagonal
- Minorminer was used to map on our abstract maps and on the full Pegasus map

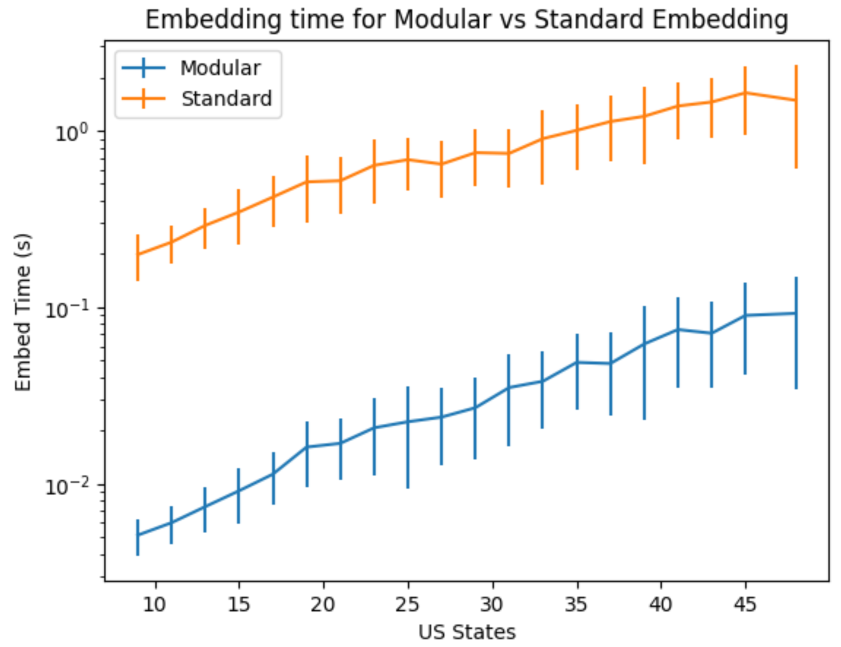
Results

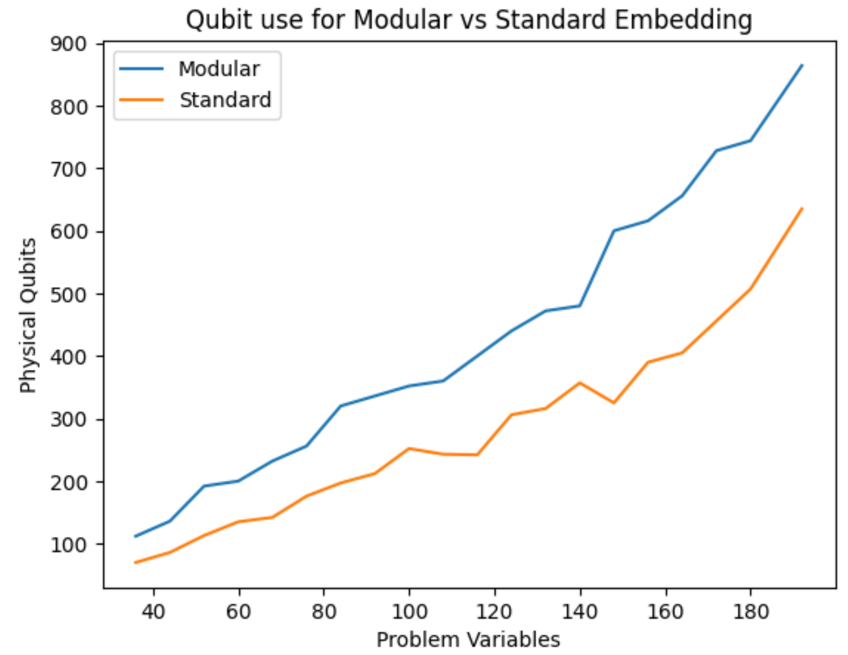
- Map color problem based on US continental map, starting with Tennessee
- Compared metrics of the mappings: Average/Max chain length, total number of qubits, embedding time
  - Modular (blue) performed worse than standard (red), except in embed time
- Ran some problems on physical machines, compared correctness
  - Modular mappings once again underperformed when compared to standard mapping

Number of States	Virtual Qubits	% Correct	Physical Qubits	Av. Chain Length	Max Length	Embed Time (s)
9	36	14	112	3.111	4	0.0051
11	44	6	136	3.091	6	0.0060
15	60	1	200	3.333	8	0.0091
19	76	0	256	3.368	6	0.0161
25	100		352	3.52	6	0.0224
35	140		480	3.429	8	0.0486
48	192		864	4.5	12	0.0922

Number of States	Virtual Qubits	% Correct	# Qubits	Av. Chain Length	Max Length	Embed Time (s)
9	36	25	70	1.944	3	0.1984
11	44	22	86	1.955	2	0.2333
15	60	5	122	2.033	4	0.3123
19	76	4	176	2.358	4	0.5123
25	100	1	252	2.520	5	0.6845
35	140	0	357	2.412	5	1.0033
48	192	0	635	3.307	9	1.4851

TABLE: Selected results for 4 color maps of US.
Top: Modular embedding. Bottom: Standard embedding.
3 color, clique cover, and DWave Chimera architecture performed similarly.





Conclusions

- Finding good modular maps is non-trivial
  - Modular embedding is faster but worse than full map embedding in this case
  - Modular embedding is better in other situations [1]
  - Likely better performance with cell size 2 or different necessary connections between cells
  - Unable to take advantage of many connections on Pegasus

References/Acknowledgements

https://github.com/lanl/NchooseK

[1] Joseph Fustero, Scott Palmtag, and Frank Mueller "Quantum Annealing Stencils with Applications to Fuel Loading of a Nuclear Reactor" in *IEEE International Conference on Quantum Computing and Engineering (QCE)*, Oct 2021

[2] Ellis Wilson, Frank Mueller, and Scott Pakin "Combining hard and soft constraints in quantum constraint-satisfaction systems" In *Proceedings of the International Conference on High Performance Computing, Networking, Storage and Analysis (SC '22).* IEEE Press, 2022, Article 13, 1–14.

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